Çankaya University – ECE Department – ECE 474 (MT)

Student Name : Date : 08.04.2013
Student Number : Open book exam, Duration : 2 Hours

Questions

1. (70 Points) a) A fibre can be operated at what is called zero dispersion wavelength of λ_{ZD} . Fig. 1 displays the position of λ_{ZD} together with material dispersion parameter D_M , waveguide dispersion parameter D_W and total dispersion parameter D . If a single mode fibre with $V = 2.2$, $n_1 = 1.46$, $\Delta = 0.4$ percent is operated at wavelength of λ_{ZD} , using the graphs given in Figs. 1-3, calculate the amount of intramodal dispersion that occurs in this fibre length of $L = 10$ km with source spectral width of $\sigma_\lambda = 0.7$ nm . Estimate the maximum fibre capacity in Mbits / second.
b) For the same single mode fibre, calculate the percentage of power that propagates in the core.
c) For the same single mode fibre, read the attenuation in dB / km, i.e. $\alpha(\text{dB / km})$ from Fig. 4 , convert this into attenuation coefficient α_p, find the received power if the power launched into the fibre is $P(z=0) = 100\ \mu\text{W}$. Evaluate the resulting SNR at the receiver if the fibre is used at its maximum capacity.
d) Evaluate and plot the NA of the same single mode fibre.

Solution : a) From Fig. 1, we read $\lambda_{ZD} = \lambda = 1.31\ \mu\text{m}$. Thus

$$f = \frac{c}{\lambda} = \frac{3\times10^{8}}{1.31\times10^{-6}} = 2.29\times10^{14}\ \text{Hz} \quad , \quad \omega = 2\pi f = 1.44\times10^{15}\ \text{rad/sec} \tag{1.1}$$

Using Fig. 2 of this exam paper and (4.3.12) of Attenuation and dispersion in fibres_March 2013_HTE

$$\frac{dn}{d\lambda} \simeq \frac{(1.462-1.461)/3}{(1.6-1.31)\times10^{-6}} = 1149 \simeq \frac{dn_{2g}}{d\lambda}$$

$$\text{Material dispersion parameter}: D_M = \frac{1}{c}\frac{dn_{2g}}{d\lambda} = 3.831\times10^{-6}\ \text{s/m}^2$$

$$= 3.831\times10^{-6} \overbrace{\times10^{12}}^{\text{for conversion into psec}} \overbrace{\times10^{3}}^{\text{for conversion into km}^{-1}} \overbrace{\times10^{-9}}^{\text{for conversion into nm}^{-1}} = 3.831\ \text{ps/km/nm} \tag{1.2}$$

From Fig. 1, we read $D_M \approx 3$ ps/km/nm , quite in agreement. Waveguide dispersion parameter on the other hand

$$\text{Waveguide dispersion parameter}: D_W = -\frac{2\pi\Delta}{\lambda^2}\left[\frac{n_{2g}^2}{n_2\omega}V\frac{d^2(Vb_n)}{dV^2} + \frac{dn_{2g}}{d\omega}\frac{d(Vb_n)}{dV}\right] \tag{1.3}$$

By using the part of (4.3.4) of Attenuation and dispersion in fibres_March 2013_HTE, we convert from the derivative with respect to wavelength into radial frequency as follows

$$\frac{d}{d\omega} = -\frac{\lambda^2}{2\pi c}\frac{d}{d\lambda} \tag{1.4}$$

Then, inserting numeric values in (1.3) and using Fig. 3, we get

$$\begin{aligned} D_W &= -\frac{2\pi\times 0.4\times 10^{-2}}{\left(1.31\times 10^{-6}\right)^2}\left[\frac{\left(1.462\right)^2}{1.4542\times 1.44\times 10^{15}}\times \overbrace{0.13}^{\text{read from Fig.3}} - \frac{\left(1.31\times 10^{-6}\right)^2\times 1149}{2\pi\times 3\times 10^8}\overbrace{0.97}^{\text{read from Fig.3}}\right] \\ &= -1.928\times 10^{-6}\ \text{ s/m}^2 = -1.928\ \text{ ps/km/nm} \end{aligned} \tag{1.5}$$

From Fig. 1, we read $D_W \approx -2$ ps/km/nm.

For total dispersion and fibre bandwidth, maximum bit rate that can be offered with $L = 10$ km and $\sigma_\lambda = 0.7$ nm

$$\begin{aligned} &\delta T = \left(D_M + D_W\right)L\sigma_\lambda = DL\sigma_\lambda = 10\times 1.903\times 0.7 = 13.3 \text{ ps} \\ &B_t = \frac{1}{\delta T} = 75.2 \text{ GHz : Total bandwidth offered over 10 km} \\ &C_t \simeq B_t = 75.2 \text{ Gb/s : Total capacity offered over 10 km} \end{aligned} \tag{1.6}$$

b) For percentage of power propagating in the core, from (4.4) of Notes on Fibre Propagation_Jan 2013_HTE, we get

$$\Gamma_c = \frac{P_{core}}{P_{total}} = 1 - \exp\left(-\frac{2}{w_s^2}\right) = 1 - \exp\left[-\frac{2}{\left(0.65 + 1.619V^{-1.5} + 2.879V^{-6}\right)^2}\right] = 0.7671 \tag{1.7}$$

c) At $\lambda_{ZD} = \lambda = 1.31\,\mu\text{m}$, from Fig. 4, we read $\alpha = 0.35$ dB/km, then using (3.3) of Attenuation and dispersion in fibres_March 2013_HTE, we have

$$\alpha_p = \frac{\alpha}{4.343} = 0.0806 \text{ km}^{-1} \tag{1.8}$$

After using (3.1) of the same notes,

$$P_r = P\left(z = 10 \text{ km}\right) = \exp\left(-\alpha_p L\right)P\left(z = 0\right) = 44.67\,\mu\text{W} \tag{1.9}$$

For SNR calculations, we benefit from (4.1.11) of the same notes, hence

$$T_a\,(\text{absolute temprature}) = 273 + 20 = 293\ {}^0\text{K}$$

$$S_n(f) = \frac{kT_a}{2} = \frac{1.38\times10^{-23}\times293}{2} = 2.0217\times10^{-21}\ \text{J : Two side noise spectral density}$$

$$P_n = 2S_n(f)B_t = 2\times2.0217\times10^{-21}\times75.2\times10^{9} = 3.04\times10^{-10}\ \text{W} = 3.04\times10^{-4}\ \mu\text{W: Noise power}$$

$$\text{SNR (at receiver)} = \frac{P_r}{P_n} = \frac{44.67\ \mu\text{W}}{3.04\times10^{-4}\ \mu\text{W}} = 1.47\times10^{5}\ \rightarrow 51.67\ \text{dB} \qquad (1.10)$$

d) According to (2.4) of Notes on Fibre Propagation_Jan 2013_HTE, NA of the fibre is given by

$$n_2 = n_1(1-\Delta) = 1.46\times\left(1-0.4\times10^{-2}\right) = 1.4542$$

$$\text{NA} = \sin(\theta_{0c}) = n_1\sin(\theta_c) = \left(n_1^2 - n_2^2\right)^{0.5} = \left[(1.46)^2 - (1.4542)^2\right]^{0.5} = 0.13$$

$$\theta_{0c} = \sin^{-1}(\text{NA}) = 7.47^0 \qquad (1.11)$$

The related plot can be found in Fig. 1.1.

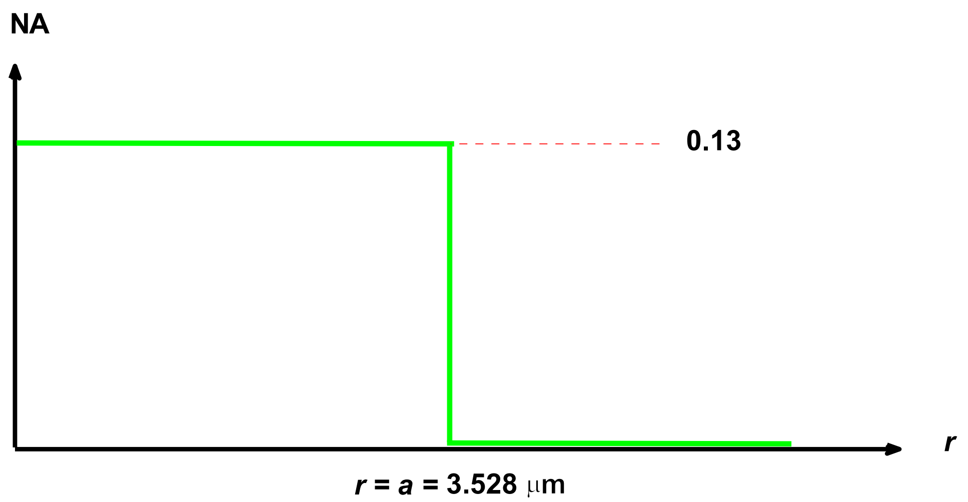

Fig. 1.1 Plot of NA for the given single mode fibre.

Note that fibre radius is calculated from

$$a = \frac{V}{k\left(n_1^2 - n_2^2\right)^{0.5}} = \frac{V}{\frac{2\pi}{\lambda}\left(n_1^2 - n_2^2\right)^{0.5}} = \frac{2.2}{\frac{2\pi}{1.31\times10^{-6}}\left[(1.46)^2 - (1.4542)^2\right]} = 3.528\ \mu\text{m} \qquad (1.12)$$

2. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer.

a) Single mode fibre is obtained by observing the V, i.e., normalized frequency value for TE_{01} and TM_{01} : True, for single mode operation, we select V to be below the roots of TE_{01} and TM_{01} at $w_n = 0$.

b) In graded index fibres, ray propagation takes place according to Fermat's principle : True, according to (2.1) of Notes on Propagation in GI fibres_Feb 2013_HTE.

c) In single mode fibres, rays can be classified as meridional and skew : False. In single mode fibres, radius of the core is comparable in size to the wavelength of the light source, so the mode theory applies, thus propagation cannot be described in terms of rays.

d) In single mode fibres, attenuation is less than multimode fibres : False, there is no such distinction, since both single and multimode fibres are made up of the same material.

e) In single mode fibres, dispersion is less than multimode fibres : True, in single mode fibres, intermodal dispersion, arising from the existence of many modes is absent

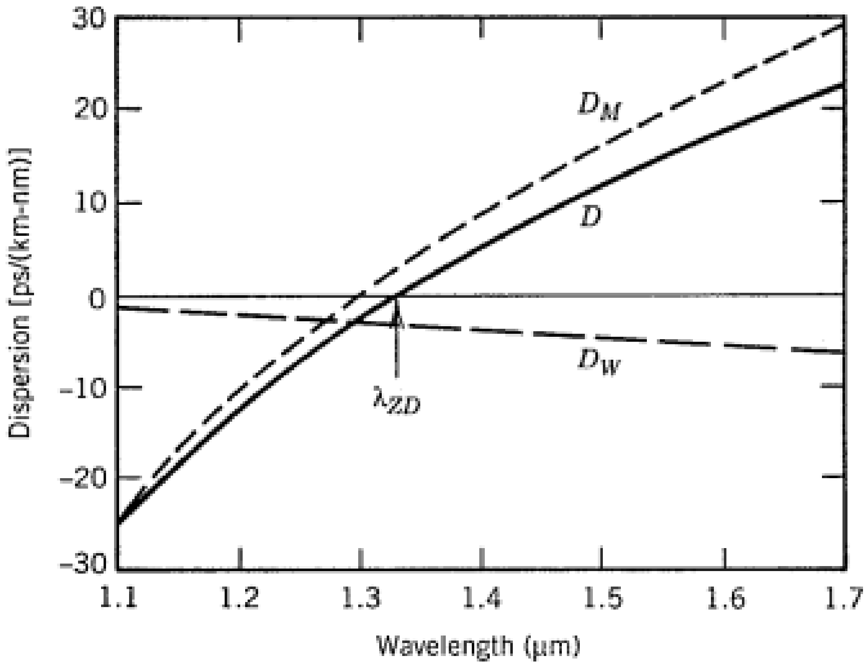

Fig. 1 Curves of D, D_M and D_W, showing the position of λ_{ZD} (Fig. 2.10 of Agrawal).

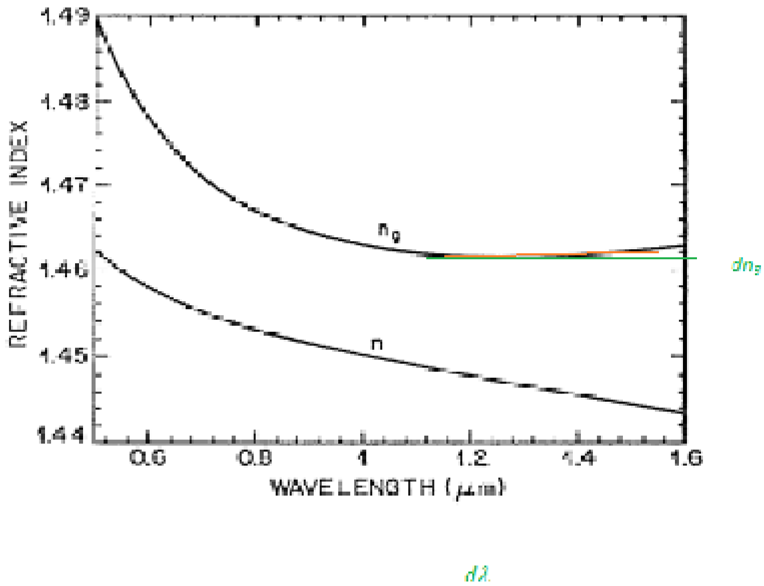

Fig. 2 Variation of refractive index with wavelength (Fig. 2.8 of Agrawal).

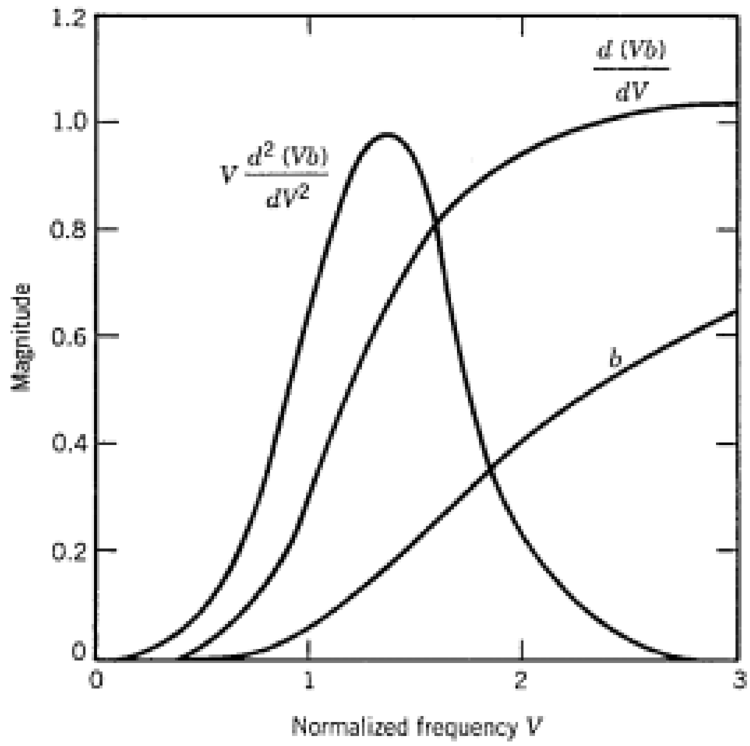

Fig. 3 Variations of b_n, $\frac{d(Vb_n)}{dV}$ and $\frac{d^2(Vb_n)}{dV^2}$ with V. Note that in our notation b is b_n (Fig. 2.9 of Agrawal).

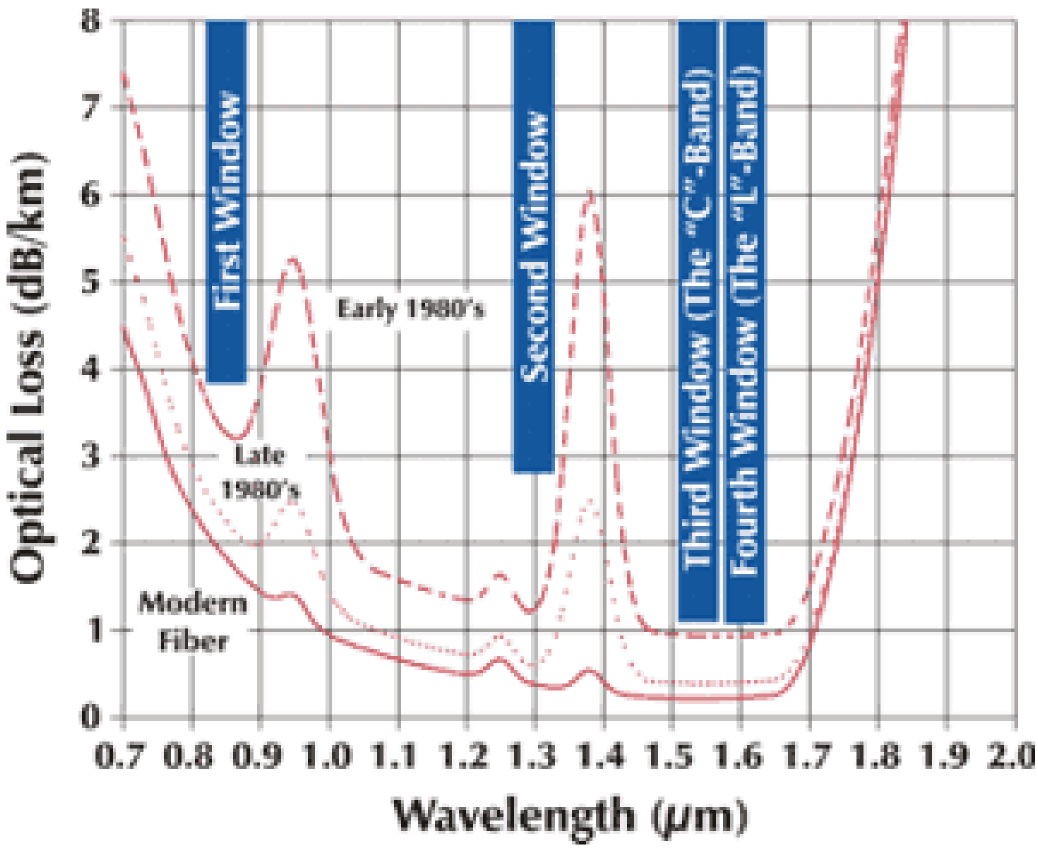

Fig. 4 Variation of fibre attenuation with wavelength.