Çankaya University – ECE Department – ECE 474 (FE)

Student Name : Date : 03.06.2014
Student Number : Open book exam, Duration : 2 Hours

Questions

1. (70 Points) A Gaussian source beam with $\alpha_s = 2$ cm emits light at $\lambda = 1.55\ \mu$m. For the following three cases, find $\alpha_B,\ \alpha_f,\ \alpha_r,\ F_r,\ z_B,\ z_{R_1},\ z_{R_2}$ at $z = 0.5$ km, 1 km, 5 km. Make plots of beam size α_r, and radius of curvature parameter F_r, variations against z for the three cases.

a) $F_s = 500$ m

b) $F_s = -500$ m

c) $F_s \to \infty$

Classify the beam types, in a), b) c). Find the on-axis, i.e., $r = 0$ intensity for the three cases at $z = 0.5$ km, 1 km, 5 km.

Solution : a) This is a convergent beam. Using (G24), (G28), (G29), (G30), (P6) of Notes on free space propagation for ECE 474_Nisan 2012, we get

$$\alpha_B = \left(\frac{4\alpha_s^2 F_s^2}{4F_s^2 + k^2\alpha_s^4}\right)^{1/2} = 1.05 \text{ cm} \ ,\ \alpha_f = \frac{2F_s}{k\alpha_s} = 1.23 \text{ cm} \ ,\ z_B = \frac{k^2\alpha_s^4 F_s}{4F_s^2 + k^2\alpha_s^4} \simeq 363.23 \text{ m independent of } z$$

$$z_{R_1} = \frac{k^2\alpha_s^4 F_s - 2k\alpha_s^2 F_s^2}{4F_s^2 + k^2\alpha_s^4} = 138.83 \text{ m} \ ,\ z_{R_2} = \frac{k^2\alpha_s^4 F_s + 2k\alpha_s^2 F_s^2}{4F_s^2 + k^2\alpha_s^4} = 585.62 \text{ m} \quad \text{independent of } z$$

$$F_r = -\frac{k^2\alpha_s^4 F_s^2 - 2k^2\alpha_s^4 F_s z + 4F_s^2 z^2 + k^2\alpha_s^4 z^2}{4F_s^2 z - k^2\alpha_s^4 F_s + k^2\alpha_s^4 z} = -F_s = -500 \text{ m} \ ,\ \alpha_r = \alpha_f = 1.23 \text{ cm at } z = 0.5 \text{ km} \tag{1.1}$$

$$F_r = -\frac{k^2\alpha_s^4 F_s^2 - 2k^2\alpha_s^4 F_s z + 4F_s^2 z^2 + k^2\alpha_s^4 z^2}{4F_s^2 z - k^2\alpha_s^4 F_s + k^2\alpha_s^4 z} = -716 \text{ m}$$

$$\alpha_r = \left(\frac{k^2\alpha_s^4 F_s^2 - 2k^2\alpha_s^4 F_s z + 4F_s^2 z^2 + k^2\alpha_s^4 z^2}{k^2\alpha_s^2 F_s^2}\right)^{1/2} = 3.18 \text{ cm} \quad \text{at } z = 1 \text{ km} \tag{1.2}$$

$$F_r = -4648 \text{ m} \ ,\ \alpha_r = 21.82 \text{ cm at } z = 5 \text{ km} \tag{1.3}$$

b) Since this is a divergent beam, we have

$$\alpha_B = \alpha_s = 2 \text{ cm} \quad \alpha_f \ ,\ z_B \ ,\ z_{R_1} \ ,\ z_{R_2} \text{ are undefined}$$

$$F_r = -920.1 \text{ m} \ ,\ \alpha_r = 4.19 \text{ cm at } z = 0.5 \text{ km} \tag{1.4}$$

$$F_r = -1399 \text{ m} \ ,\ \alpha_r = 6.49 \text{ cm at } z = 1 \text{ km} \tag{1.5}$$

$$F_r = -5371 \text{ m} \ ,\ \alpha_r = 25.22 \text{ cm at } z = 5 \text{ km} \tag{1.6}$$

c) This is a collimated beam, so we have

$$\alpha_B = \alpha_s = 2 \text{ cm} \quad \alpha_f \quad , \; z_B = 0 \, , \; z_R = 0.5k\alpha_s^2 = 810.7 \text{ m}$$

$$F_r = -1815 \text{ m} \;\; , \;\; \alpha_r = 2.35 \text{ cm at } z = 0.5 \text{ km} \tag{1.6}$$

$$F_r = -1657 \text{ m} \;\; , \;\; \alpha_r = 3.18 \text{ cm at } z = 1 \text{ km} \tag{1.7}$$

$$F_r = -5131 \text{ m} \;\; , \;\; \alpha_r = 12.5 \text{ cm at } z = 5 \text{ km} \tag{1.8}$$

The above calculations are performed in the Matlab file FEQ1_29052013.m.

The plots of beam size α_r, and radius of curvature parameter F_r, variations against z are shown in Figs. 1.1

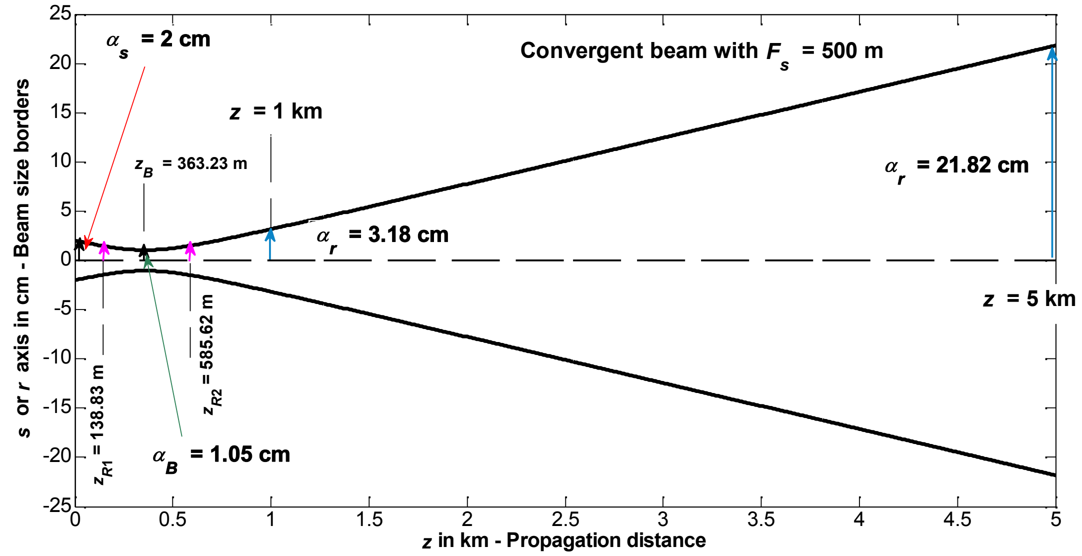

Fig. 1.1 Beam size α_r, variation against z for $F_s = 500$ m.

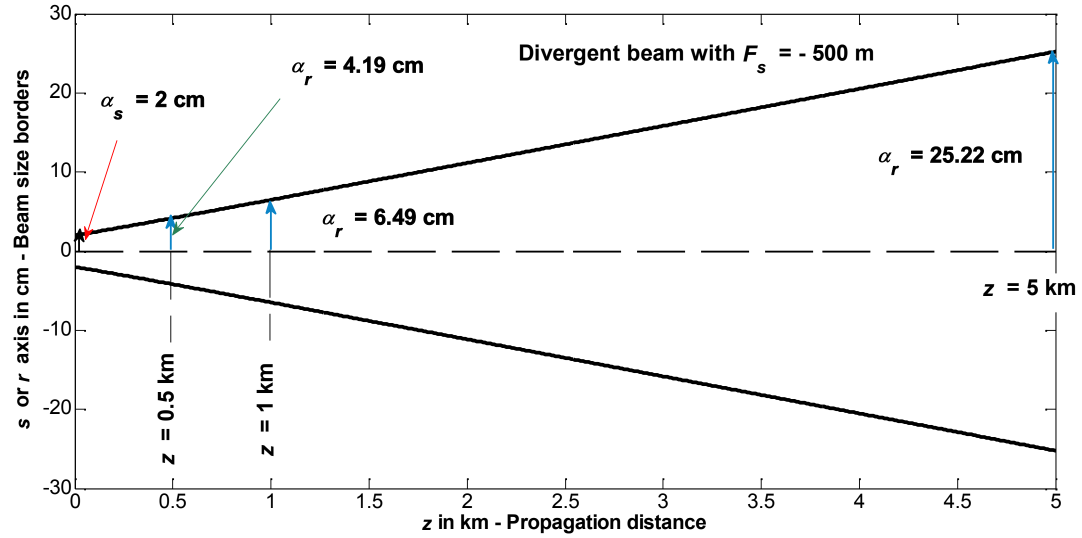

Fig. 1.2 Beam size α_r, variation against z for $F_s = -500$ m.

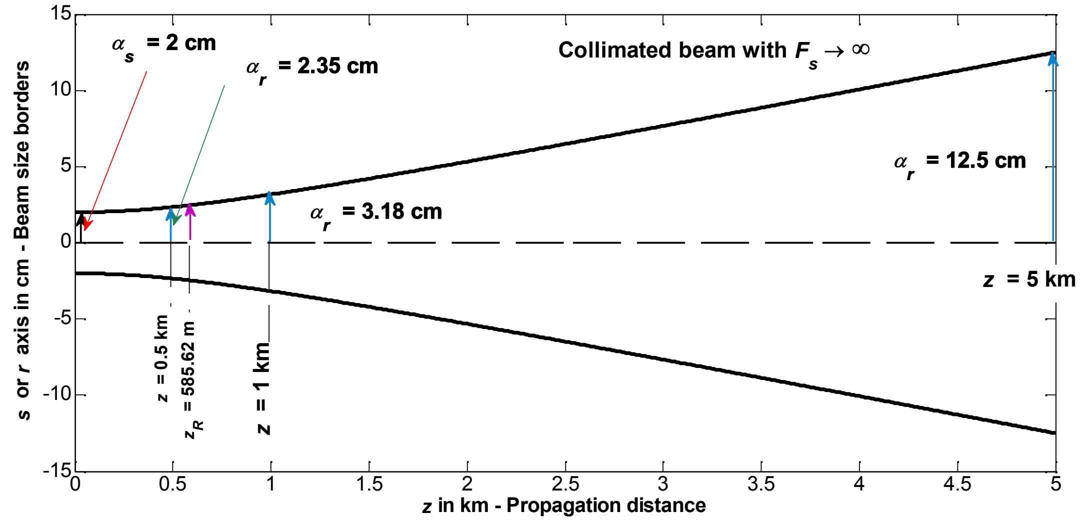

Fig. 1.3 Beam size α_r, variation against z for $F_s \to \infty$.

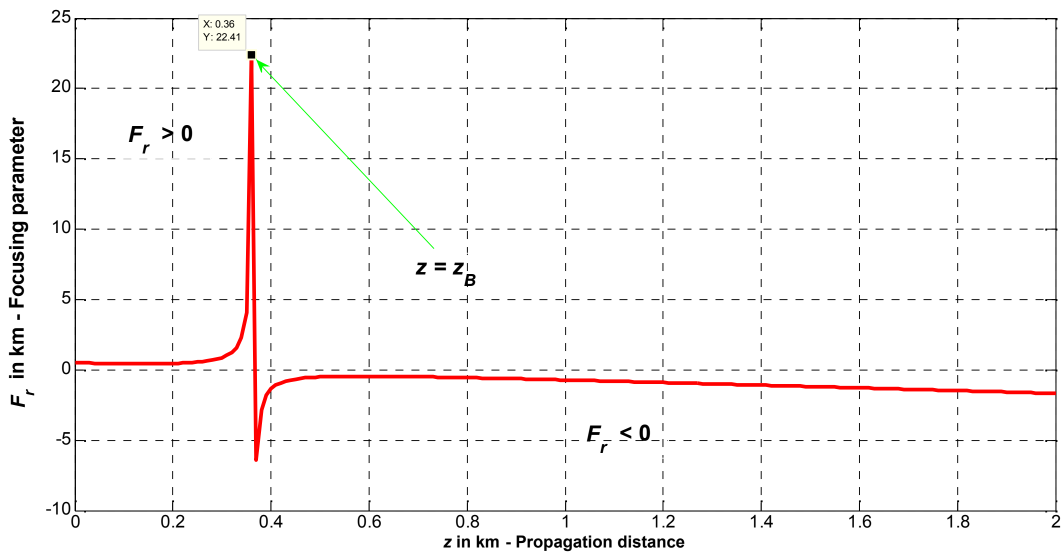

Fig. 1.4 radius of curvature parameter F_r, variation against z for $F_s = 500$ m.

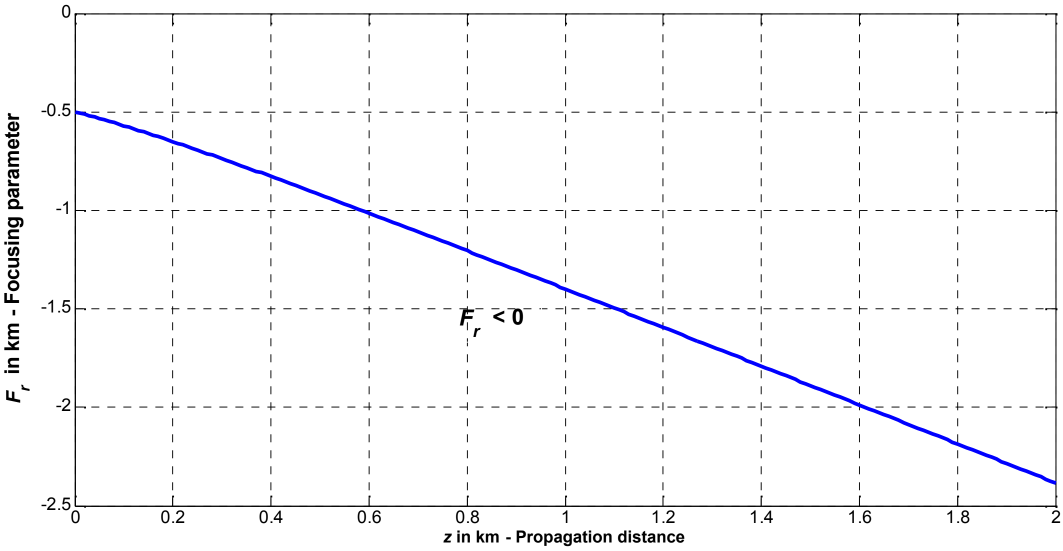

Fig. 1.5 radius of curvature parameter F_r, variation against z for $F_s = -500$ m.

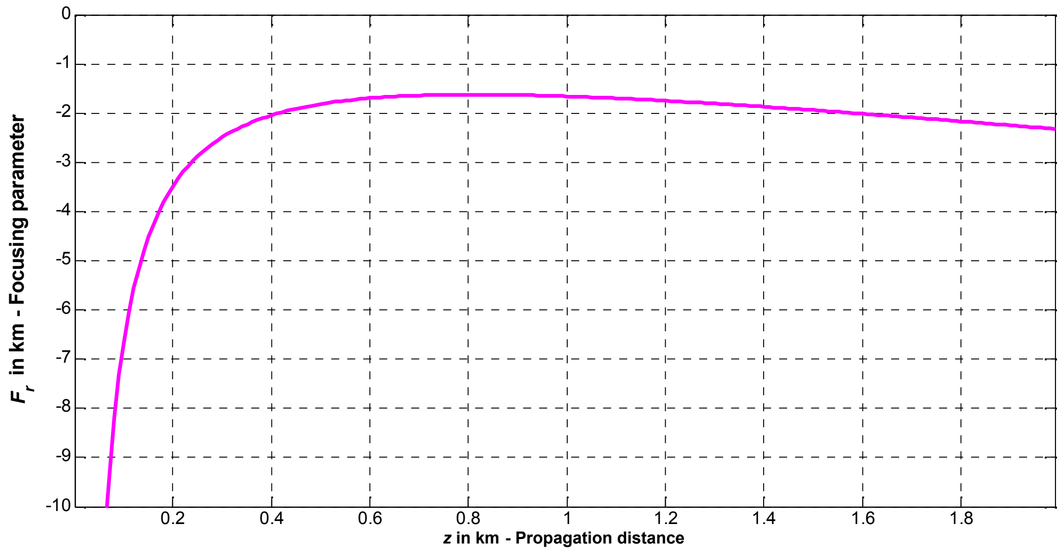

Fig. 1.6 radius of curvature parameter F_r, variation against z for $F_s \to \infty$.

2. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer.

a) In a PIN photodiode, the diode current generated I_p is directly proportional to reflectivity coefficient, R_f : From (4.3) of lecture notes entitled, "Optical transmitters and receivers_2013_HTE", we have

$$I_p = \frac{q}{hf} P_0 \left[1 - \exp\left(-\alpha_s w\right)\right]\left(1 - R_f\right) \qquad (4.3)$$

Therefore I_p is inversely proportional to reflectivity coefficient, R_f .

b) In an laser resonator, the number of modes created is directly proportional to the cavity length, L : Althogh from (3.10) of lecture notes entitled, "Optical transmitters and receivers_2013_HTE", it seems that the index of the mode is directly proportional to cavity length, in physical reality the profile of the gain parameter that determines the number of modes as explained in the same notes.

c) The responsivity of a photodiode is wavelength dependent : True, as seen from Fig. 4.3 of lecture notes entitled, "Optical transmitters and receivers_2013_HTE".

d) Diffraction takes place, when an optical beam propagates in atmosphere : True, as explained above (G12) of lecture notes entitled, "Notes on free space propagation for ECE 474_Nisan 2012".

e) Helmholtz equation is derived from paraxial wave equation : False, considering the equation flow from (B1) to (B6) in lecture notes entitled, "Notes on free space propagation for ECE 474_Nisan 2012".

f) Paraxial wave equation is used, when propagation is confined along one axis : True, as explained underneath (B5) in lecture notes entitled, "Notes on free space propagation for ECE 474_Nisan 2012".