Çankaya University – ECE Department – ECE 474 (FE)

Student Name : Date : 05.06.2012
Student Number : Open book exam, Duration : 2 Hours

Questions

1. (40 Points) Find the source size or sizes (α_s) for a) Convergent beam with the focusing parameter $F_s = 1.1$ km and b) Collimated beam, that will give a beam size of $\alpha_r = 11$ cm at $z = 4$ km away from the source plane. For these three beams, find Rayleigh range $(z_R,\ z_{R_1},\ z_{R_2})$, beam waist size (α_B), beam waist location (z_B), radius of curvature (F_r) at $z = 4$ km and beam size at focus (α_f) at all source sizes. For these three beams, plot beam size variation along the propagation axis, labelling the appropriate points along this axis.

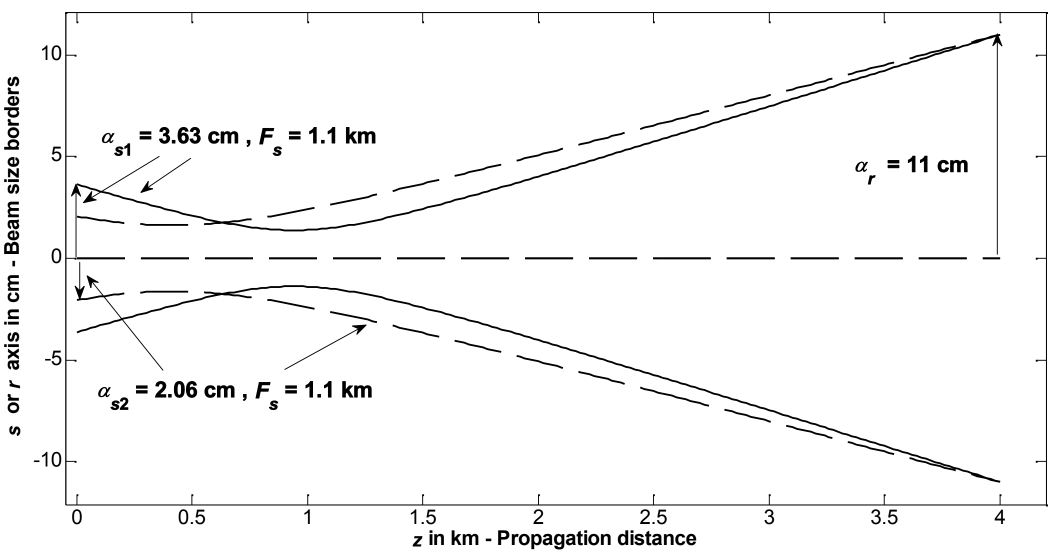
Solution : By using (G24) of Notes on free space propagation for ECE 474_Nisan 2012, we have

$$\alpha_r = \left(\frac{k^2\alpha_s^4F_s^2 - 2k^2\alpha_s^4F_sz + 4F_s^2z^2 + k^2\alpha_s^4z^2}{k^2\alpha_s^2F_s^2} \right)^{1/2}$$

a) After rearranging the above as one sided equation, we get

$$\left(k^2F_s^2 - 2k^2F_sz + k^2z^2\right)\alpha_s^4 - k^2\alpha_r^2F_s^2\alpha_s^2 + 4F_s^2z^2 = 0$$

By inserting $\alpha_r = 11$ cm, $F_s = 1.1$ km, $z = 4$ km, $\lambda = 1550$ nm for convergent beam, the solutions are $\alpha_{s1} = 3.63$ cm and $\alpha_{s2} = 2.06$ cm . This is illustrated below



Additionally for this beam (again using the appropriate equations from the same notes) we have for Rayleigh range $\left(z_{R_1},\, z_{R_2}\right)$

$$z_{R_1}\left(\alpha_s=\alpha_{s1}=3.63\text{ cm}\right)=\frac{k^2\alpha_s^4F_s-2k\alpha_s^2F_s^2}{4F_s^2+k^2\alpha_s^4}=533.11\text{ m}$$
$$z_{R_2}\left(\alpha_s=\alpha_{s1}=3.63\text{ cm}\right)=\frac{k^2\alpha_s^4F_s+2k\alpha_s^2F_s^2}{4F_s^2+k^2\alpha_s^4}=1.328\text{ km}$$
$$z_{R_1}\left(\alpha_s=\alpha_{s2}=2.06\text{ cm}\right)=\frac{k^2\alpha_s^4F_s-2k\alpha_s^2F_s^2}{4F_s^2+k^2\alpha_s^4}=-116.406\text{ m}$$
$$z_{R_2}\left(\alpha_s=\alpha_{s2}=2.06\text{ cm}\right)=\frac{k^2\alpha_s^4F_s+2k\alpha_s^2F_s^2}{4F_s^2+k^2\alpha_s^4}=951.127\text{ m}$$

For beam waist size $\left(\alpha_B\right)$

$$\alpha_B\left(\alpha_s=\alpha_{s1}=3.63\text{ cm}\right)=\left(\frac{4\alpha_s^2F_s^2}{4F_s^2+k^2\alpha_s^4}\right)^{1/2}=1.38\text{ cm}$$
$$\alpha_B\left(\alpha_s=\alpha_{s2}=2.06\text{ cm}\right)=\left(\frac{4\alpha_s^2F_s^2}{4F_s^2+k^2\alpha_s^4}\right)^{1/2}=1.62\text{ cm}$$

For beam waist location $\left(z_B\right)$

$$z_B\left(\alpha_s=\alpha_{s1}=3.63\text{ cm}\right)=\frac{k^2\alpha_s^4F_s}{4F_s^2+k^2\alpha_s^4}=940.46\text{ m}$$
$$z_B\left(\alpha_s=\alpha_{s2}=2.06\text{ cm}\right)=\frac{k^2\alpha_s^4F_s}{4F_s^2+k^2\alpha_s^4}=417.36\text{ m}$$

For radius of curvature $\left(F_r\right)$ at $z=4$ km

$$F_r\left(\alpha_s=\alpha_{s1}=3.63\text{ cm}, z=4\text{ km}\right)=-\frac{k^2\alpha_s^4F_s^2-2k^2\alpha_s^4F_sz+4F_s^2z^2+k^2\alpha_s^4z^2}{4F_s^2z-k^2\alpha_s^4F_s+k^2\alpha_s^4z}=-3.109\text{ km}$$
$$F_r\left(\alpha_s=\alpha_{s2}=2.06\text{ cm}, z=4\text{ km}\right)=-\frac{k^2\alpha_s^4F_s^2-2k^2\alpha_s^4F_sz+4F_s^2z^2+k^2\alpha_s^4z^2}{4F_s^2z-k^2\alpha_s^4F_s+k^2\alpha_s^4z}=-3.66\text{ km}$$

For beam size at focus $\left(\alpha_f\right)$

$$\alpha_f\left(\alpha_s=\alpha_{s1}=3.63\text{ cm}\right)=\frac{2F_s}{k\alpha_s}=1.5\text{ cm}\ ,\ \alpha_f\left(\alpha_s=\alpha_{s2}=2.06\text{ cm}\right)=\frac{2F_s}{k\alpha_s}=2.63\text{ cm}$$

b) We repeat above calculations, for Collimated beam, that is at $F_s\to\infty$

$$\alpha_r =_{F_s \to \infty} \left(\frac{k^2 \alpha_s^4 F_s^2 - 2k^2 \alpha_s^4 F_s z + 4F_s^2 z^2 + k^2 \alpha_s^4 z^2}{k^2 \alpha_s^2 F_s^2} \right)^{1/2} = \left(\frac{k^2 \alpha_s^4 + 4z^2}{k^2 \alpha_s^2} \right)^{1/2}$$

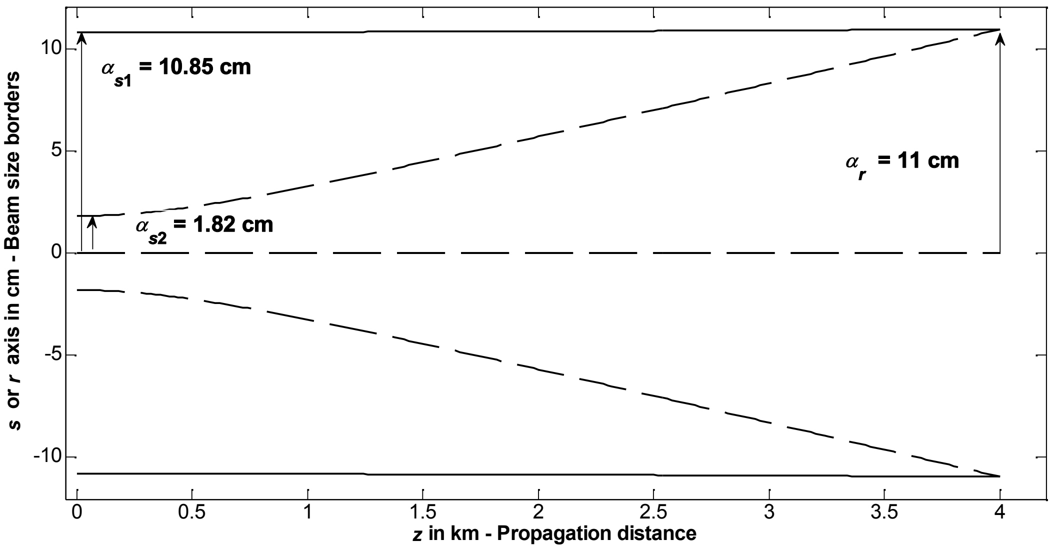
After rearrangement, this gives the following one sided equation,

$$k^2 \alpha_s^4 - k^2 \alpha_r^2 \alpha_s^2 + 4z^2 = 0$$

where the solutions are

$$\alpha_{s1,2}^2 = \frac{k^2 \alpha_r^2 \pm \sqrt{k^4 \alpha_r^4 - 16k^2 z^2}}{2k^2}$$

By inserting $\alpha_r = 11$ cm, $z = 4$ km, $\lambda = 1550$ nm, the solutions are $\alpha_{s1} = 10.85$ cm and $\alpha_{s2} = 1.82$ cm. This is illustrated below



we have for Rayleigh range (z_R)

$$z_R(\alpha_s = \alpha_{s1} = 10.85 \text{ cm}) = 0.5k\alpha_s^2 = 23.86 \text{ km} \;,\; z_R(\alpha_s = \alpha_{s2} = 1.82 \text{ cm}) = 0.5k\alpha_s^2 = 671.39 \text{ m}$$

For beam waist size (α_B)

$$\alpha_B(\alpha_s = \alpha_{s1} = 10.85 \text{ cm}) = \lim_{F_s \to \infty} \left(\frac{4\alpha_s^2 F_s^2}{4F_s^2 + k^2 \alpha_s^4} \right)^{1/2} = \alpha_{s1} = 10.85 \text{ cm}$$

$$\alpha_B(\alpha_s = \alpha_{s2} = 1.82 \text{ cm}) = \lim_{F_s \to \infty} \left(\frac{4\alpha_s^2 F_s^2}{4F_s^2 + k^2 \alpha_s^4} \right)^{1/2} = \alpha_{s2} = 1.82 \text{ cm}$$

For beam waist location (z_B)

$$z_B\left(\alpha_s = \alpha_{s1} = 10.85\text{ cm}\right) = \lim_{F_s\to\infty}\left(\frac{k^2\alpha_s^4 F_s}{4F_s^2 + k^2\alpha_s^4}\right) \to 0$$

$$z_B\left(\alpha_s = \alpha_{s2} = 1.82\text{cm}\right) = \lim_{F_s\to\infty}\left(\frac{k^2\alpha_s^4 F_s}{4F_s^2 + k^2\alpha_s^4}\right) \to 0$$

For radius of curvature $\left(F_r\right)$ at $z = 4$ km

$$F_r\left(\alpha_s = \alpha_{s1} = 10.85\text{ cm}\right) = \lim_{F_s\to\infty} -\frac{k^2\alpha_s^4 F_s^2 - 2k^2\alpha_s^4 F_s z + 4F_s^2 z^2 + k^2\alpha_s^4 z^2}{4F_s^2 z - k^2\alpha_s^4 F_s + k^2\alpha_s^4 z} = -\frac{k^2\alpha_s^4 + 4z^2}{4z} = -146.33\text{ km}$$

$$F_r\left(\alpha_s = \alpha_{s2} = 1.82\text{ cm}\right) = \lim_{F_s\to\infty} -\frac{k^2\alpha_s^4 F_s^2 - 2k^2\alpha_s^4 F_s z + 4F_s^2 z^2 + k^2\alpha_s^4 z^2}{4F_s^2 z - k^2\alpha_s^4 F_s + k^2\alpha_s^4 z} = -\frac{k^2\alpha_s^4 + 4z^2}{4z} = -4.113\text{ km}$$

Beam size at focus $\left(\alpha_f\right)$ for collimated beam is meaningless since there is no focusing.

2. (30 Points) A photo diode has a reflectivity coefficient of $R_f = 0.15$, depletion region width of $w = 1$ mm, absorption coefficient of $\alpha_s = 1$ mm^{-1}. Find the photo diode current (I_p) generated for an incident optical power of $P_0 = 1$ nW at $\lambda = 1550$ nm. Find the efficiency (η) and responsivity (R) of this photo diode.

Solution : From Eq. (6-4) of Gerd Keiser, "Optical Fiber Communications" 3nd Ed. 2000, McGraw Hill book, we have

$$I_p = \frac{q}{h\nu} P_0 \left[1 - \exp(-\alpha_s w)\right]\left(1 - R_f\right)$$

where the terms are as given in the question. Then I_p will become

$$I_p = \frac{1.6\times10^{-19} \times 1\times10^{-9}\left[1 - \exp(-1)\right](1 - 0.15)}{6.625\times10^{-34}\left(3\times10^{8}/1.55\times10^{-6}\right)} = 0.67 \text{ nA}$$

From Eq. (6-5) of the same book

$$\eta = \frac{I_p / q}{P_0 / h\nu} = \left[1 - \exp(-\alpha_s w)\right]\left(1 - R_f\right) = 0.5373$$

And from Eq. (6-6) of the same book

$$R = \frac{I_p}{P_0} = \left[1 - \exp(-\alpha_s w)\right]\left(1 - R_f\right) = 0.67$$

3. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer.

a) In single mode fibre, dispersion is determined by core radius : In single mode fibre dispersion consists of two separate components, 1) Material dispersion which is related to refractive index, so has no relation to the core radius of the fibre, 2) Waveguide dispersion which is related V (normalized frequency parameter), which is in turn related to fibre core radius.

b) Single mode fibre offers more bandwidth than multimode fibre : True, since we do not have intermodal dispersion in single mode fibre.

c) Waveguide dispersion and material dispersion are different : True as explained in a). Material dispersion is basically due to the variation of refractive index with wavelength. So to the source, there appears to be more than one fibre. Waveguide dispersion on the other hand is the variation of fibre V with respect to wavelength, this way, there appears to be many sources exiting the same fibre.

d) If we use LEDs as light sources, less dispersion occurs in fibres : False, LEDs compared with lasers generate more dispersion.

e) In plane wave, radius of curvature is finite : False, in plane wave the equi phase surfaces are just vertical lines, therefore, radius of curvature is infinite there.