Çankaya University – ECE Department – ECE 474 (FE)

Student Name : Date : 29.05.2013
Student Number : Open book exam, Duration : 2 Hours

Questions

1. (65 Points) A Gaussian source beam with $F_s = 800$ m, $\lambda = 1.55\ \mu$m is received as $F_r = -1.5$ km at $z = 2$ km away from the source. Identify the beam type. Find the followings

a) Source size, α_s and beam size, α_r at $z = 4$ km,

b) Beam waist size, α_B, beam waist location, z_B, beam size at focus, α_f,

c) Rayleigh range, z_{R_1}, z_{R_2}.

Write the field and intensity expressions of this beam at source plane, $z = 0$ and at receiver plane, $z = 2$ km.

Plot the beam size, α_r and the radius of curvature, F_r variations in the propagation range of $z = 0 \rightarrow 5$ km.

Solution : a) Using (G28) of Notes on free space propagation for ECE 474_Nisan 2012, which is

$$F_r = -\frac{k^2\alpha_s^4 F_s^2 - 2k^2\alpha_s^4 F_s z + 4F_s^2 z^2 + k^2\alpha_s^4 z^2}{4F_s^2 z - k^2\alpha_s^4 F_s + k^2\alpha_s^4 z} \tag{1.1}$$

and setting it to zero, such that

$$F_r + \frac{k^2\alpha_s^4 F_s^2 - 2k^2\alpha_s^4 F_s z + 4F_s^2 z^2 + k^2\alpha_s^4 z^2}{4F_s^2 z - k^2\alpha_s^4 F_s + k^2\alpha_s^4 z} = 0 \tag{1.2}$$

By inserting the numeric values given in the question, from the plot of the LHS in FEQ1_29052013.m, we find the zero crossing in terms of α_s as $\alpha_s = 2.5645$ cm . Now by using (G24) of Notes on free space propagation for ECE 474_Nisan 2012, we find beam size, α_r as

$$\alpha_r = \left(\frac{k^2\alpha_s^4 F_s^2 - 2k^2\alpha_s^4 F_s z + 4F_s^2 z^2 + k^2\alpha_s^4 z^2}{k^2\alpha_s^2 F_s^2}\right)^{1/2} = 5.44 \text{ cm at } z = 2 \text{ km}$$

$$= 12.82 \text{ cm at } z = 4 \text{ km} \tag{1.3}$$

b) From (G30), (G29), (P6) of Notes on free space propagation for ECE 474_Nisan 2012, and using the Matlab file FEQ1_29052013.m, we find

$$\alpha_B = \left(\frac{4\alpha_s^2 F_s^2}{4F_s^2 + k^2\alpha_s^4}\right)^{1/2} = 1.32 \text{ cm} \quad , \quad z_B = \frac{k^2\alpha_s^4 F_s}{4F_s^2 + k^2\alpha_s^4} = 588.2741 \text{ m} \quad , \quad \alpha_f = \frac{2F_s}{k\alpha_s} = 1.54 \text{ cm} \tag{1.4}$$

c) From (G31) of Notes on free space propagation for ECE 474_Nisan 2012, and using the Matlab file FEQ1_29052013.m, we find

$$z_{R_1} = \frac{k^2\alpha_s^4 F_s - 2k\alpha_s^2 F_s^2}{4F_s^2 + k^2\alpha_s^4} = 235.3537 \text{ m} \quad , \quad z_{R_2} = \frac{k^2\alpha_s^4 F_s + 2k\alpha_s^2 F_s^2}{4F_s^2 + k^2\alpha_s^4} = 941.1946 \text{ m} \qquad (1.5)$$

From (G26) and (G27) of Notes on free space propagation for ECE 474_Nisan 2012, the fields on source and receiver planes, respectively at $z = 0$ and $z = 2$ km are

$$U_s(s,\phi_s) = A_c \exp\left(-k\alpha s^2\right) = A_c \exp\left(-\frac{s^2}{\alpha_s^2} - \frac{jks^2}{2F_s}\right)$$

$$= A_c \exp\left[-\frac{s^2}{\left(2.5645\times10^{-2}\right)^2} - \frac{j\times4.0537\times10^6\times s^2}{2\times800}\right] \qquad (1.5)$$

$$U_r(r,\phi_r,z) = A_c \frac{\exp(jkz)}{1+2j\alpha z}\exp\left(-\frac{r^2}{\alpha_r^2} - j\frac{kr^2}{2F_r}\right)$$

$$= A_c \frac{\exp\left(j\times4.0537\times10^6\times2\times10^3\right)}{1+2j\times\left[\frac{1}{4.0537\times10^6\times\left(2.5645\times10^{-2}\right)^2} + \frac{j}{2\times800}\right]\times2\times10^3}$$

$$\times\exp\left[-\frac{r^2}{\left(2.5645\times10^{-2}\right)^2} + \frac{j\times4.0537\times10^6\times r^2}{2\times1500}\right] \qquad (1.6)$$

$$I_r(r,\phi_r,z) = \frac{A_c^2\alpha_s^2}{\alpha_r^2}\exp\left(-\frac{2r^2}{\alpha_r^2}\right) = A_c^2\frac{\left(2.5645\times10^{-2}\right)^2}{\left(5.44\times10^{-2}\right)^2}\exp\left[-\frac{2r^2}{\left(5.44\times10^{-2}\right)^2}\right] \qquad (1.7)$$

The plots of the beam size, α_r and the radius of curvature, F_r variations in the propagation range of $z = 0 \rightarrow 5$ km are shown in Figs. 1.1 and 1.2.

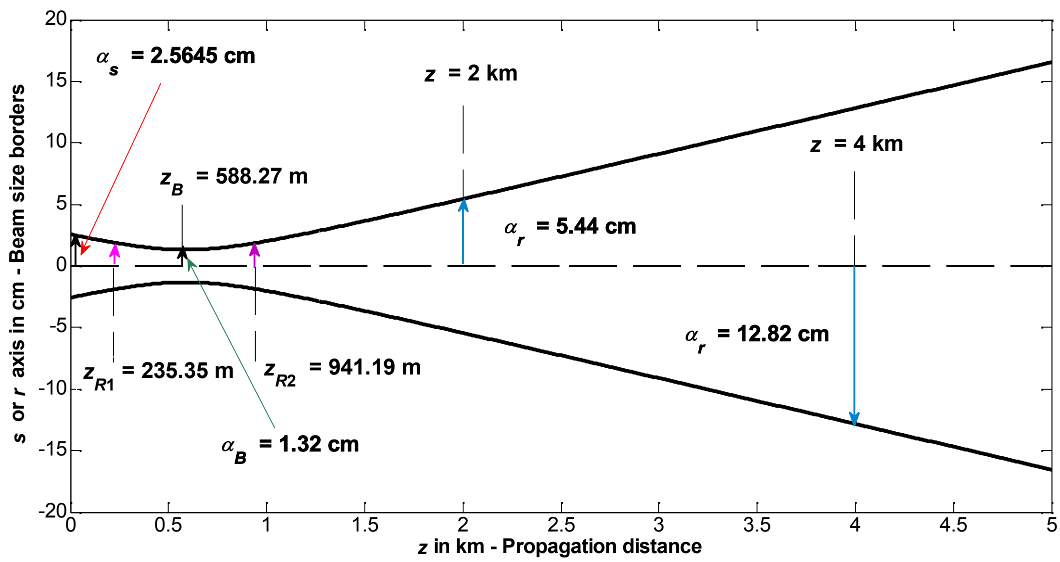

Fig. 1.1 The variation of the beam size, α_r against the propagation range of $z = 0 \rightarrow 5$ km.

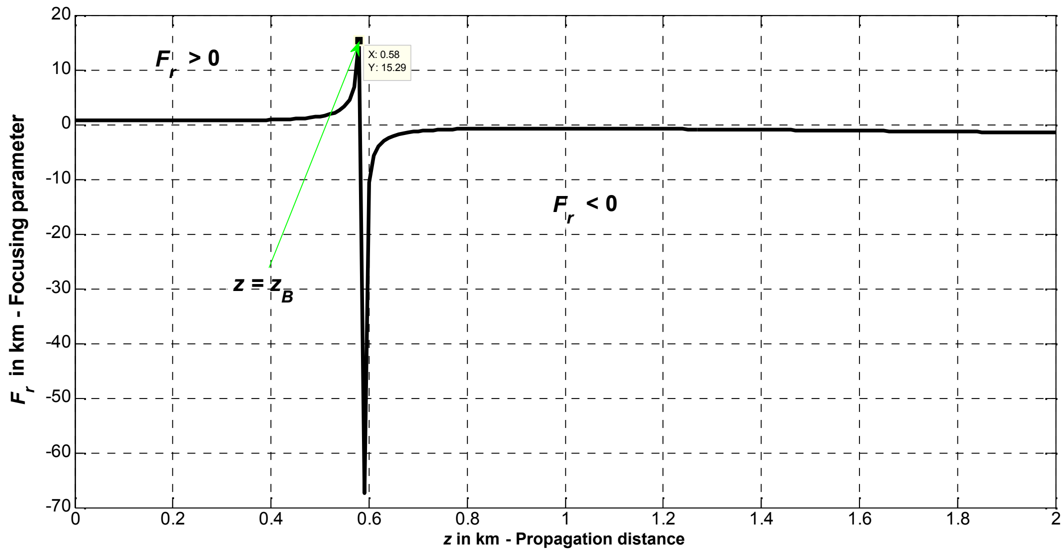

Fig. 1.2 The variations of the radius of curvature, F_r against the propagation range of $z = 0 \rightarrow 5$ km.

2. (35 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer.

a) For a ternary alloy of $\text{Ga}_{1-x}\text{Al}_x\text{As}$, if $x = 0.07$, the wavelength of operation is $\lambda = 0.83\ \mu\text{m}$: From (2.2) of Optical transmitters and receivers_2013_HTE, we have for a ternary alloy, the energy gap formulation of

$$E_g = 1.424 + 1.266x + 0.266x^2 \qquad (2.1)$$

Inserting $x = 0.07$, we get

$$E_g = 1.424 + 1.266 \times 0.07 + 0.266 \times 0.07^2 = 1.51\ \text{eV} \qquad (2.2)$$

Hence, the wavelength of operation found to be

$$\lambda(\mu\text{m}) = \frac{1.24}{E_g(\text{eV})} = \frac{1.24}{1.51} = 0.82\ \mu\text{m} \qquad (2.3)$$

Therefore, the answer is false.

b) If a PIN photodiode has a reflectivity coefficient of $R_f = 0.12$, depletion region width of $w = 1.2$ mm, absorption coefficient of $\alpha_s = 1\ \text{mm}^{-1}$ and receives an incident optical power of $P_0 = 1.5$ nW at $\lambda = 1310$ nm, then its efficiency will become $\eta = 72\ \%$ and the responsivity $R = 0.41$: From (4.3), (4.4) and (4.6) of Optical transmitters and receivers_2013_HTE, we have

$$\eta = \frac{I_p / q}{P_0 / hf} = \left[1 - \exp(-\alpha_s w)\right](1 - R_f) = 0.6149$$

$$R = \frac{q}{hf}\left[1 - \exp(-\alpha_s w)\right](1 - R_f) = 0.6485 \qquad (2.4)$$

Therefore, the answer is false.

c) In graded index fibres, ray path is helical and only material dispersion exists : From Figs. 1.2, 2.5 and 2.7 in Notes on Propagation in GI fibres_Feb 2013_HTE, we see that in graded index fibres, ray path is helical, but from Attenuation and dispersion in fibres_March 2013_HTE, all types of dispersion (including material) exist in graded index fibre.

d) The responsivity of an avalance photodiode is greater than a PIN photodiode : According to section 5 of Optical transmitters and receivers_2013_HTE, we can only speak about multiplication factor in an avalanche photodiode.

e) In single mode fibres, part of the field propagates in the core, the remaining part becomes attenuated in the cladding : False, in single mode fibre every care is taken so that the cladding material has the same attenuation characteristics as the core, so that the extension of the field in the cladding suffers the same attenuation as that in the core.