Çankaya University – ECE Department – ECE 474 (Final)

Student Name : Date : 02.06.2009
Student Number : Open book exam

Questions

1. (70 Points) A Gaussian beam focused at 1 km, has a beam diameter of $2\alpha_r = 36$ cm at the propagation distance of z = 5 km. Assuming $\lambda = 1.55\ \mu$m, find the followings

a) What type of a beam is this ?, i.e. collimated, convergent, divergent ?

b) Source size of beam at transmitter plane, i.e. α_s and phase front radius of curvature at transmitter plane, i.e. F_s

c) Phase front radius of curvature at z = 5 km, i.e. F_r .

d) If this beam is now focused at 5 km, calculate α_f (beam size at focus) and F_r at this point.

e) When focused at 1 km and 5 km, respectively find beam waist location and beam waist size, i.e. z_B and α_B .

Solution : a) Obviously, this is a convergent beam with $F_s = 1$ km.

b) From Notes on free space propagation for ECE 474_Nisan 2012, using (G24) we have

$$\alpha_r^2 = \frac{k^2\alpha_s^4 F_s^2 - 2k^2\alpha_s^4 F_s z + 4F_s^2 z^2 + k^2\alpha_s^4 z^2}{k^2\alpha_s^2 F_s^2}$$

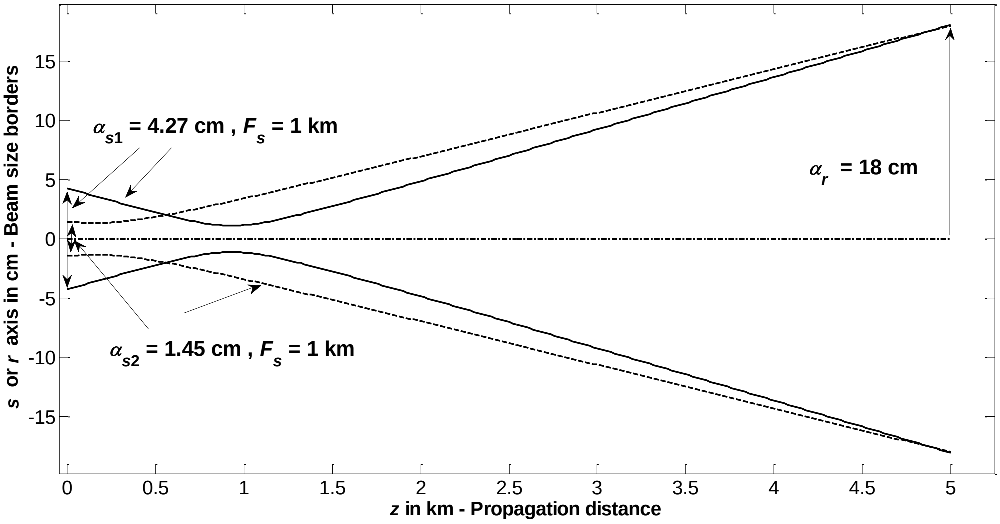
By collecting terms on one side and converting the above into a semi-quadratic equation we get

$$\left(k^2F_s^2 - 2k^2F_s z + k^2z^2\right)\alpha_s^4 - k^2\alpha_r^2F_s^2\alpha_s^2 + 4F_s^2z^2 = 0$$

By inserting $\alpha_r = 18$ cm, $k = 2\pi/\lambda = 4.05367\text{x}10^6$, $z = 5$ km and $F_s = 1$ km, then solving the above equation, we get the following values

$\alpha_{s1} = 4.27$ cm , $\alpha_{s2} = 1.45$ cm

Both beams are physically possible, as illustrated below. From this point onwards, we continue by taking $\alpha_s = \alpha_{s1} = 4.27$ cm



Phase front radius of curvature at transmitter plan is simply $F_s = 1\,\text{km}$.

c) By using (G28) of the same notes we get

$$F_r = -\frac{k^2\alpha_s^4F_s^2 - 2k^2\alpha_s^4F_s z + 4F_s^2z^2 + k^2\alpha_s^4z^2}{4F_s^2z - k^2\alpha_s^4F_s + k^2\alpha_s^4z} = -4\text{ km}$$

d) If the beam is now focused at $F_s = 5\text{ km}$, then according to (P6) of the notes, with the insertion of $z = F_s = 5\text{ km}$, we have

$$\alpha_f = \left(\frac{k^2\alpha_s^4F_s^2 - 2k^2\alpha_s^4F_s^2 + 4F_s^4 + k^2\alpha_s^4F_s^2}{k^2\alpha_s^2F_s^2}\right)^{1/2} = \frac{2F_s}{k\alpha_s} = 2.89\text{ cm}$$

e) For z_B and α_B, we respectively use (G29) and (G30) of the notes which is

$$z_B = \frac{k^2\alpha_s^4F_s}{4F_s^2 + k^2\alpha_s^4} \quad , \quad \alpha_B = \left(\frac{4\alpha_s^2F_s^2}{4F_s^2 + k^2\alpha_s^4}\right)^{1/2}$$

Thus

For $F_s = 1\text{ km}$, $z_B = 931.77\text{ m}$, $\alpha_B = 0.11\text{ cm}$
For $F_s = 5\text{ km}$, $z_B = 1.766\text{ km}$, $\alpha_B = 0.34\text{ cm}$

2. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer

a) A laser has a broader emission spectrum than LED : False, on the contrary, an LED has a broader emission spectrum than a laser.

b) LED light is obtained from a resonator construction, when gains exceed losses : False, this description is valid for lasers.

c) Efficiency in a photo diode measures the ratio of number of incident photons to number of electron hole pairs generated : False, it should read that

$$\text{Efficiency} = \eta = \frac{\text{No of electron-hole pairs generated}}{\text{No of incident photons}}$$

Otherwise $\eta > 1$ and unreasonable.

d) Efficiency of a photo diode is always larger than its responsivity : False such a comparison cannot be made. Efficiency is defined in c). Responsivity on the other hand is $R = \frac{I_p}{P_0}$

e) The numerical aperture of a single mode fibre is less than graded index fibre : Generally true, since to reduce V to the limit of 2.45, core radius can be lowered or refractive index difference can be reduced, while $\text{NA} = \left(n_1^2 - n_2^2\right)^{0.5}$