Çankaya University – ECE Department – ECE 474 (Final)

Student Name : Date : 26.05.2008
Student Number : Open book exam

Questions

1. (40 Points) In free space optics propagation, beam waist size α_B is defined as the minimum beam size along propagation axis. This means that α_B can be obtained by differentiating $\alpha_r = \left(\frac{k^2\alpha_s^4 F_s^2 - 2k^2\alpha_s^4 F_s z + 4F_s^2 z^2 + k^2\alpha_s^4 z^2}{k^2\alpha_s^2 F_s^2} \right)^{1/2}$ with respect to z, where z is the distance variable from the source (transmitter) plane to receiver. Prove that this is the case, finding α_B and z_B (the propagation distance at which α_B occurs) as

$$\alpha_B = \left(\frac{4\alpha_s^2 F_s^2}{4F_s^2 + k^2\alpha_s^4} \right)^{1/2} \quad \text{and} \quad z_B = \frac{k^2\alpha_s^4 F_s}{4F_s^2 + k^2\alpha_s^4}$$

Additionally write for α_f (the beam size at $z = z_f = F_s$, with F_s being the geometric focus). Compare α_B with α_f and z_B with z_f, indicating whether $\alpha_B > \alpha_f$ or $\alpha_B < \alpha_f$ and $z_B > z_f$ or $z_B < z_f$. Give your reasons.

Solution : By taking (G24) of Notes on free space propagation for ECE 474_Nisan 2012 and rearranging it as follows

$$\alpha_r = \left(\frac{k^2\alpha_s^4 F_s^2 - 2k^2\alpha_s^4 F_s z + 4F_s^2 z^2 + k^2\alpha_s^4 z^2}{k^2\alpha_s^2 F_s^2} \right)^{1/2} = \alpha_s \left(1 - \frac{2z}{F_s} + \frac{z^2}{F_s^2} + \frac{4z^2}{k^2\alpha_s^4} \right)^{1/2}$$

Now differentiate with respect to z to get

$$\frac{\partial \alpha_r}{\partial z} = \frac{\alpha_s}{2} \left(1 - \frac{2z}{F_s} + \frac{z^2}{F_s^2} + \frac{4z^2}{k^2\alpha_s^4} \right)^{-1/2} \left(-\frac{2}{F_s} + \frac{2z}{F_s^2} + \frac{8z}{k^2\alpha_s^4} \right)$$

z_B can be obtained by setting $\frac{\partial \alpha_r}{\partial z}$ to zero, hence

$$z_B = \frac{\partial \alpha_r}{\partial z} = 0 \quad \text{or} \quad \left(-\frac{2}{F_s} + \frac{2z_B}{F_s^2} + \frac{8z_B}{k^2\alpha_s^4} \right) = 0$$

Finally we will get

$$z_B = \frac{k^2\alpha_s^4 F_s}{4F_s^2 + k^2\alpha_s^4}$$

which is the same as (G29) of the notes. Now insert $z = z_B$ in the above equation of α_r, then

$$\alpha_B = \left(\frac{k^2\alpha_s^4 F_s^2 - 2k^2\alpha_s^4 F_s z_B + 4F_s^2 z_B^2 + k^2\alpha_s^4 z_B^2}{k^2\alpha_s^2 F_s^2} \right)^{1/2} = \left(\frac{4\alpha_s^2 F_s^2}{4F_s^2 + k^2\alpha_s^4} \right)^{1/2}$$

The right most equation is the same as (G30) of the notes. To find α_f, we insert in α_r expression, $z = F_s$, which means

$$\alpha_f = \left(\frac{k^2\alpha_s^4 F_s^2 - 2k^2\alpha_s^4 F_s F_s + 4F_s^2 F_s^2 + k^2\alpha_s^4 F_s^2}{k^2\alpha_s^2 F_s^2} \right)^{1/2} = \frac{2F_s}{k\alpha_s}$$

To make a comparison α_B with α_f and z_B with z_f, we analyse the case of convergent beam, since for collimated beam $\alpha_B = \alpha_s$, thus $z_B = 0$ and $\alpha_f \to \infty$, $z_f \to \infty$. On the other hand, for divergent beam, beam waist location is behind the source plane.

Now for convergent beam, $F_s > 0$, then

$$\alpha_B^2 = \frac{4\alpha_s^2 F_s^2}{4F_s^2 + k^2\alpha_s^4} < \frac{4F_s^2}{k^2\alpha_s^2} = \alpha_f^2 \quad \text{holds true since} \quad 16F_s^4 > 0$$

For the comparison of z_B with z_f, we write as follows

$$z_B = \frac{k^2\alpha_s^4 F_s}{4F_s^2 + k^2\alpha_s^4} < z_f = F_s \quad \text{holds true since} \quad 4F_s^3 > 0$$

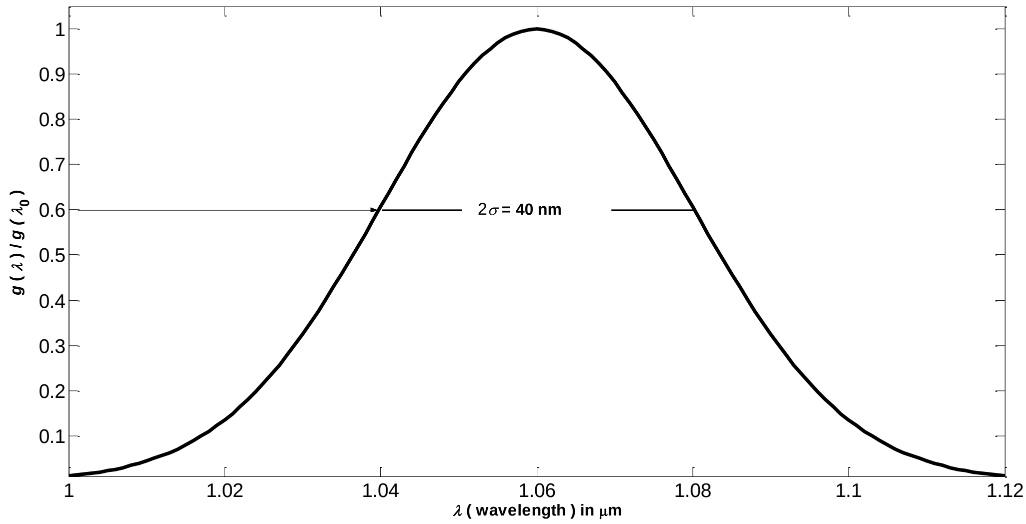
In summary for convergent beam, beam waist location is always prior to geometric focus and the beam size at beam waist is always smaller that the beam size at geometric focus.

2. (30 Points) A laser has an emission wavelength of λ_0 = 1.06 μm with a spectral width of σ = 20 nm and a peak gain of $g(0)$ = 30 cm^{-1}. Using the Gaussian spectral form, plot $g(\lambda)$ against λ. If the laser cavity is 200 μm long, and refractive index, n is 3.2, calculate how many modes will be excited in this laser. Are these modes longitudinal, lateral of transverse modes ?

Solution : Using

$$g(\lambda) = g(0)\exp\left[-\frac{(\lambda-\lambda_0)^2}{2\sigma^2}\right]$$

where $\lambda_0 = 1.06\ \mu$m, $\sigma = 20$ nm, $g(0) = 30$ cm^{-1}. The relevant plot is shown below



Note that $\lambda = \lambda_0 \pm \sigma$, $\exp\left[-\frac{(\lambda-\lambda_0)^2}{2\sigma^2}\right] = \exp(-0.5) \simeq 0.6$.

From the formula

$$m = \frac{L}{\lambda_0 / 2n}$$

where L is cavity length, λ_0 is the central wavelength of operation, n is the refractive index of lasing medium.

$$m = \frac{200\text{x}10^{-6}}{1.06\text{x}10^{-6}/(2\text{x}3.2)} \simeq 1207 \text{ modes are excited.}$$

These are longitudinal modes, since they are L (laser cavity) dependent.

3. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer

a) A Gaussian beam at source plane is described by Hermite polynomials of order higher than 1 : False, source beam formulation of a given Gaussian beam is

$$U_r\left(r,\phi_r\right)=A(z)\exp\left[-\frac{k\alpha r^2}{p(z)}\right] \quad \text{in cylindrical coordinates}$$

b) A divergent beam has $F_s > 0$ and its geometric focus occurs behind the source plane : The second part is true, while the first part is false, since for a divergent beam, $F_s < 0$.

c) Dispersion in a fibre increases with distance and wavelength : The first part is true, while the second part is false, since dispersion rises or falls with wavelength depending on the wavelength region considered.

d) APD has a multiplication factor, M, larger than unity : True, since

$$M=\frac{I_M}{I_p}$$

e) Responsivity of a PIN diode is the ratio of incident power to diode current : False, since it is just the reverse of this definition, i.e.,

$$\text{Responsivity}=R=\frac{I_p}{P_0}=\frac{\text{Photodiode current}}{\text{Incident optical power}}$$