Çankaya University – ECE Department – ECE 474 (MT - YO)

Student Name : Date : 04.08.2014
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Questions

1. (70 Points) Find the minimum wavelengths of single mode operation for the following two fibres

a) $V_1 = 2.3,\ a = 3.5\ \mu\text{m},\ {}_1n_1 = 1.49,\ \Delta = 5\times10^{-3},\ L_1 = 30\ \text{km}$
b) $V_2 = 2,\ a = 3.5\ \mu\text{m},\ {}_2n_1 = 1.48,\ \Delta = 5\times10^{-3},\ L_2 = 40\ \text{km}$

Assume that both fibres are driven by an optical source at the minimum wavelength of operation with a spectral width of $\sigma_\lambda = 0.2\ \text{nm}$, then by using the graphs given in Figs. 1-4, evaluate the followings for each fibre

A. The amount of total dispersion comprising material and waveguide dispersions,
B. The maximum bit rate that can be offered by each fibre,
C. The minimum source power, $P(z=0)$ that has to be launched into each fibre so that $P(z=L) = 5\ \mu\text{W}$ can be obtained at the receiver side, find the corresponding SNR,
D. The amount of power propagating in the core regions,
E. The value of the propagation constant, β for HE_{11} mode in each case.

Solution : a) We find the minimum wavelengths λ_1 and λ_2 , ${}_1n_2$ and ${}_2n_2$ for the two cases as follows

$$a)\ \lambda_1 = \frac{2\pi\,{}_1n_1 a}{V_1}(2\Delta)^{0.5} = 1.4246\ \mu\text{m}\ ,\ {}_1n_2 = {}_1n_1(1-2\Delta)^{0.5} = 1.4825$$
$$b)\ \lambda_2 = \frac{2\pi\,{}_2n_1 a}{V_2}(2\Delta)^{0.5} = 1.6273\ \mu\text{m}\ ,\ {}_2n_2 = {}_2n_1(1-2\Delta)^{0.5} = 1.4726 \qquad (1.1)$$

A. To evaluate the total dispersion comprising material and waveguide dispersions, we perform the following calculations

At $\lambda_1 = 1.4246\ \mu\text{m}$

$$f_1 = \frac{c}{\lambda_1} = \frac{3\times10^8}{1.4246\times10^{-6}} = 2.1059\times10^{14}\ \text{Hz}\quad ,\quad \omega_1 = 2\pi f_1 = 1.3231\times10^{15}\ \text{rad/sec} \qquad (1.2)$$

Using Fig. 2 of this exam paper or by reading it from the graphical output of the m file RefindexVcurves, we have

$$\left.\frac{dn}{d\lambda}\right|_{\lambda_1=1.425\ \mu\text{m}} = 4066 \simeq \frac{dn_{2g}}{d\lambda}$$

$$\text{Material dispersion parameter}: D_{M_1} = \frac{1}{c}\frac{dn_{2g}}{d\lambda} = 13.6\times10^{-6}\ \text{s/m}^2$$

$$= 13.6\times10^{-6} \overbrace{\times10^{12}}^{\text{for conversion into psec}} \overbrace{\times10^{3}}^{\text{for conversion into km}^{-1}} \overbrace{\times10^{-9}}^{\text{for conversion into nm}^{-1}} = 13.6\ \text{ps/km/nm} \quad (1.3)$$

From Fig. 2.20 of Agrawal, we read approximately the same numeric value.

For waveguide dispersion parameter, we have

$$\text{Waveguide dispersion parameter}: D_W = -\frac{2\pi\Delta}{\lambda^2}\left[\frac{n_{2g}^2}{n_2\omega}V\frac{d^2(Vb_n)}{dV^2} + \frac{dn_{2g}}{d\omega}\frac{d(Vb_n)}{dV}\right] \quad (1.4)$$

By using the part of (4.3.4) of Attenuation and dispersion in fibres_March 2013_HTE, we convert from the derivative with respect to wavelength into radial frequency as follows

$$\frac{d}{d\omega} = -\frac{\lambda^2}{2\pi c}\frac{d}{d\lambda} \quad (1.5)$$

Then, inserting numeric values in (1.4) and using Fig. 3, we get

$$D_{W_1} = -\frac{2\pi\times5\times10^{-3}}{\left(1.4246\times10^{-6}\right)^2}\left[\frac{(1.462)^2}{1.4825\times1.3231\times10^{15}}\times\overbrace{0.04}^{\text{read from Fig.3}} - \frac{\left(1.4246\times10^{-6}\right)^2\times4066}{2\pi\times3\times10^8}\overbrace{1.04}^{\text{read from Fig.3}}\right]$$
$$= -6.0425\times10^{-7}\ \text{s/m}^2 \simeq -0.604\ \text{ps/km/nm} \quad (1.6)$$

From Fig. 2.20 of Agrawal, we read slightly higher numeric value.

For total dispersion with $L_1 = 30$ km and $\sigma_\lambda = 0.2$ nm, we have

$$\delta T_1 = \left(D_{M_1} + D_{W_1}\right)L_1\sigma_\lambda = D_1 L_1 \sigma_\lambda = 12.996\times30\times0.2 = 77.98\ \text{ps} \quad (1.7)$$

Following the same steps for the second fibre, we have

$$\lambda_2 = 1.6273\ \mu\text{m} \quad , \quad f_2 = 1.8435\times10^{14}\ \text{Hz} \quad , \quad \omega_2 = 1.1583\times10^{15}\ \text{rad/sec} \quad (1.8)$$

$$\left.\frac{dn}{d\lambda}\right|_{\lambda_2=1.63\ \mu\text{m}} = 8155 \simeq \frac{dn_{2g}}{d\lambda} \ , \ D_{M_2} = 27.18\ \text{ps/km/nm} \quad (1.9)$$

$$D_{W_2} = -\frac{2\pi\times5\times10^{-3}}{\left(1.6273\times10^{-6}\right)^2}\left[\frac{(1.463)^2}{1.4726\times1.1583\times10^{15}}\times\overbrace{0.22}^{\text{read from Fig.3}} - \frac{\left(1.6273\times10^{-6}\right)^2\times8155}{2\pi\times3\times10^8}\overbrace{0.97}^{\text{read from Fig.3}}\right]$$
$$= -3.143\ \text{ps/km/nm} \quad (1.10)$$

$$\delta T_2 = \left(D_{M_2} + D_{W_2}\right) L_2 \sigma_\lambda = 192.3 \text{ ps} \quad (1.11)$$

B. We calculate the maximum bit rate that can be offered by the two fibres as follows

$$\begin{aligned}
&B_{t_1} = \frac{1}{\delta T_1} = 36.8 \text{ GHz : Total bandwidth offered over 30 km of fibre in a)}\\
&C_{t_1} \simeq B_{t_1} = 36.8 \text{ Gb/s : Total capacity offered over 30 km of fibre in a)}\\
&B_{t_2} = \frac{1}{\delta T_2} = 5.2 \text{ GHz : Total bandwidth offered over 40 km of fibre in b)}\\
&C_{t_2} \simeq B_{t_2} = 5.2 \text{ Gb/s : Total capacity offered over 40 km of fibre in b)} \quad (1.12)
\end{aligned}$$

C. At $\lambda_1 = 1.4246\ \mu\text{m}$, from Fig. 4, we read $\alpha_1 = 0.35$ dB/km, at $\lambda_2 = 1.4246\ \mu\text{m}$, from Fig. 4, we read $\alpha_2 = 0.25$ dB/km,

For SNR calculations, we benefit from (4.1.11) of Attenuation and dispersion in fibres_March 2013_HTE, hence

$$\begin{aligned}
&T_a\left(\text{absolute temprature}\right) = 273 + 20 = 293\ {}^0\text{K}\\
&S_n\left(f\right) = \frac{kT_a}{2} = \frac{1.38\times10^{-23}\times293}{2} = 2.0217\times10^{-21} \text{ J : Two side noise spectral density}\\
&a)\ P_{n_1} = 2S_n\left(f\right)B_{t_1} = 2\times2.0217\times10^{-21}\times36.8\times10^{9} = 1.481\times10^{-10} \text{ W}\\
&\qquad = 1.481\times10^{-4}\ \mu\text{W: Noise power} \rightarrow -68.3 \text{ dBm}\\
&\quad P_r = 5\ \mu\text{W} \rightarrow -23 \text{ dBm},\ \text{SNR(in dB)} = -23 \text{ dBm} - (-68.3 \text{ dBm}) = 45.3 \text{ dB}\\
&\quad P_1\left(z=0\right) = P_r(-23 \text{ dBm}) + \alpha_1\times L_1 = -12.5 \text{ dBm} \rightarrow 56.2\ \mu\text{W}\\
&b)\ P_{n_2} = 2S_n\left(f\right)B_{t_2} = 2\times2.0217\times10^{-21}\times5.2\times10^{9} = 2.093\times10^{-11} \text{ W}\\
&\qquad = 2.093\times10^{-5}\ \mu\text{W: Noise power} \rightarrow -76.8 \text{ dBm}\\
&\quad \text{SNR(in dB)} = -23 \text{ dBm} - (-76.8 \text{ dBm}) = 53.8 \text{ dB}\\
&\quad P_2\left(z=0\right) = P_r(-23 \text{ dBm}) + \alpha_2\times L_2 = -13 \text{ dBm} \rightarrow 50.1\ \mu\text{W} \quad (1.13)
\end{aligned}$$

D. The amount of power propagating in the core for each case is

$$\begin{aligned}
&a)\ V = 2.3,\ \frac{P_{1core}}{P_{1total}} = 1-\exp\left(-\frac{2}{w_s^2}\right) = 1-\exp\left[-\frac{2}{\left(0.65+1.619V^{-1.5}+2.879V^{-6}\right)^2}\right] = 0.79\\
&P_{1core} = 0.79\times P_{1total} = 0.79\times P_1\left(z=0\right) = 0.79\times56.2\ \mu\text{W} = 44.4\ \mu\text{W}\\
&a)\ V = 2,\ \frac{P_{2core}}{P_{2total}} = 1-\exp\left(-\frac{2}{w_s^2}\right) = 1-\exp\left[-\frac{2}{\left(0.65+1.619V^{-1.5}+2.879V^{-6}\right)^2}\right] = 0.71\\
&P_{2core} = 0.71\times P_{1total} = 0.71\times P_2\left(z=0\right) = 0.71\times50.1\ \mu\text{W} = 35.57\ \mu\text{W} \quad (1.12)
\end{aligned}$$

E. To find the value of the propagation constant, β for HE_{11} mode in each case, we benefit from the Matlab file Findingbeta.m and plot Fig.1.1 and Fig.1.2.

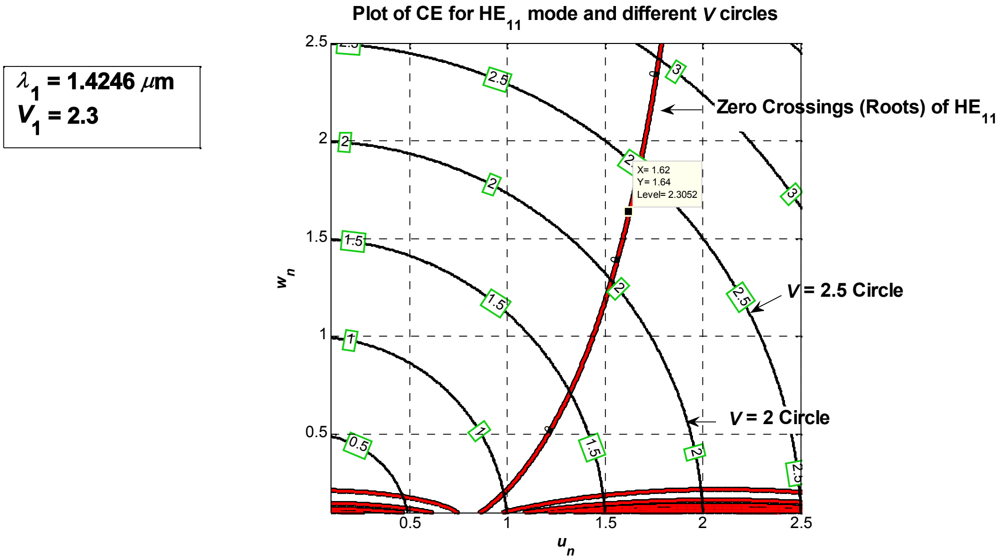

Fig. 1.1 Contour plots of the CE for HE_{11} and the associated V circles for fibre in a).

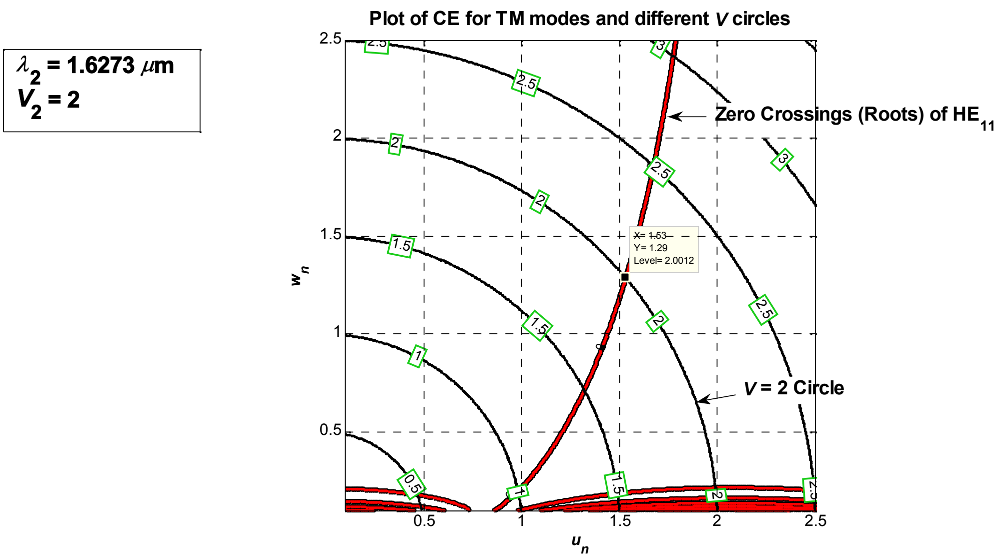

Fig. 1.1 Contour plots of the CE for HE_{11} and the associated V circles for fibre in b).

From Figs. 1.1 and 1.2, we read at the cross point of zero crossings (roots) of HE_{11} and find that for $V_1 = 2.3$ circle, $u_{n_1} = 1.62$, $w_{n_1} = 1.64$, $V_2 = 2$ circle, $u_{n_2} = 1.53$, $w_{n_2} = 1.29$. Additionally we note that

$$
{}_1k_1 = n_1 \frac{2\pi}{\lambda_1} = 6.57 \times 10^6 \quad , \quad {}_1k_2 = n_2 \frac{2\pi}{\lambda_1} = 6.538 \times 10^6
$$

$$
{}_2k_1 = n_1 \frac{2\pi}{\lambda_2} = 5.714 \times 10^6 \quad , \quad {}_2k_2 = n_2 \frac{2\pi}{\lambda_2} = 5.686 \times 10^6 \tag{1.13}
$$

Now we can calculate the propagation constants as follows

$$\beta_{n_1} = \left(a^2{}_1k_1^2 - u_{n_1}^2\right)^{0.5} = 22.94 \quad \text{or} \quad \beta_{n_1} = \left(a^2{}_1k_2^2 + w_{n_1}^2\right)^{0.5} = 22.94$$

$$\beta_1 = \frac{\beta_{n_1}}{a} = 6.5548\times10^6 \quad , \quad {}_1k_2 \le \beta_1 \le {}_1k_1 \quad \text{is satisfied}$$

$$\beta_{n_2} = \left(a^2{}_2k_1^2 - u_{n_2}^2\right)^{0.5} = 19.94 \quad \text{or} \quad \beta_{n_2} = \left(a^2{}_2k_2^2 + w_{n_2}^2\right)^{0.5} = 19.94$$

$$\beta_2 = \frac{\beta_{n_2}}{a} = 5.698\times10^6 \quad , \quad {}_2k_2 \le \beta_2 \le {}_2k_1 \quad \text{is satisfied} \qquad (1.14)$$

2. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer.

a) In graded index fibre, the projections of ray trajectories are always elliptical : False, it is elliptical for skew rays, but as straight lines for meridional rays.

b) LED sources emit incoherent light : True, the light output of LEDs have phase fluctuation along time and spatial axes.

c) Responsivity of a photodiode is always greater than unity : False, according to Fig. 4.3 of the notes entitled, "Optical transmitters and receivers_2013_HTE", responsivity cannot exceed unity.

d) Dispersion is a measure of fibre bandwidth : True, such calculations are made in ECE 474_MT_08042013_Solutions.pdf.

e) Attenuation is related to the fibre material : True, it is the fibre material, i.e, SiO_2 that determines the attenuation characteristics against wavelength.

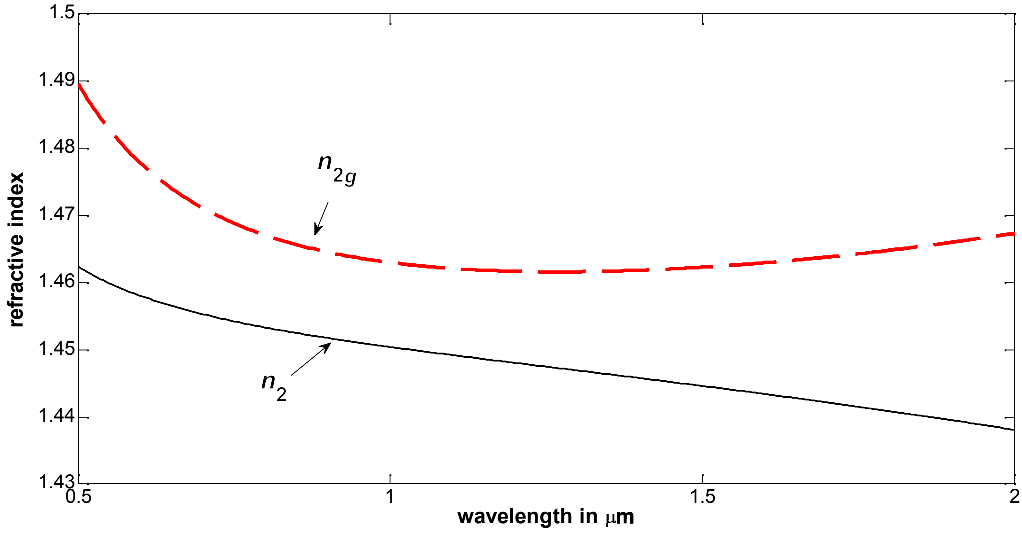

Fig. 1 Variation of cladding refractive index, n_2 and n_{2g} with wavelength.

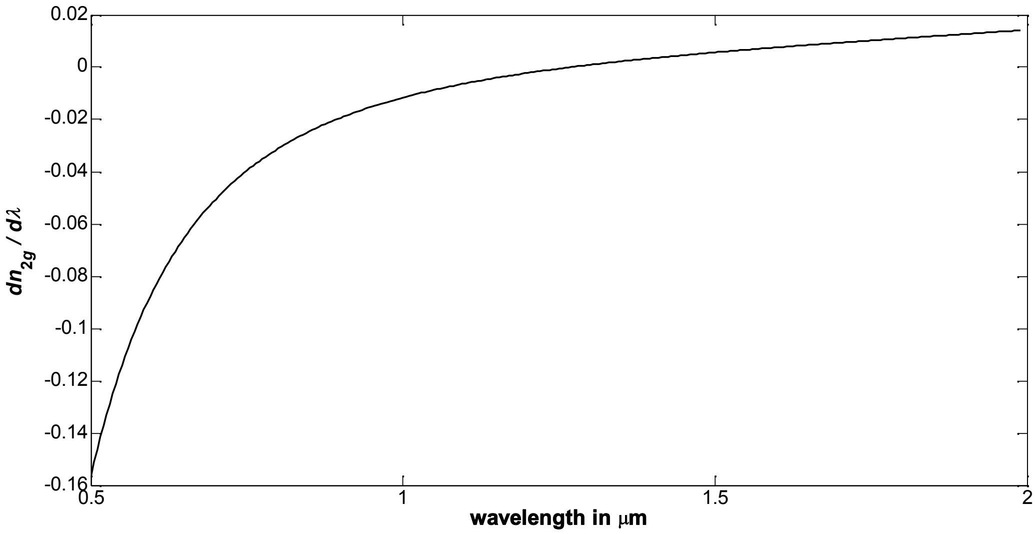

Fig. 2 Variation of the derivative of n_{2g} with wavelength.

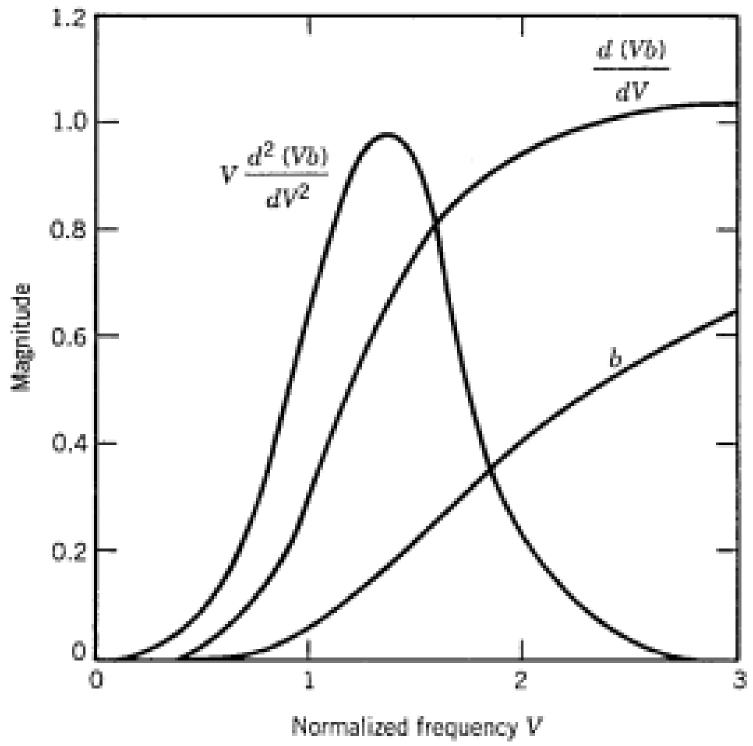

Fig. 3 Variations of b_n, $\frac{d\left(Vb_n\right)}{dV}$ and $\frac{d^2\left(Vb_n\right)}{dV^2}$ with V. Note that in our notation b is b_n (Fig. 2.9 of Agrawal).

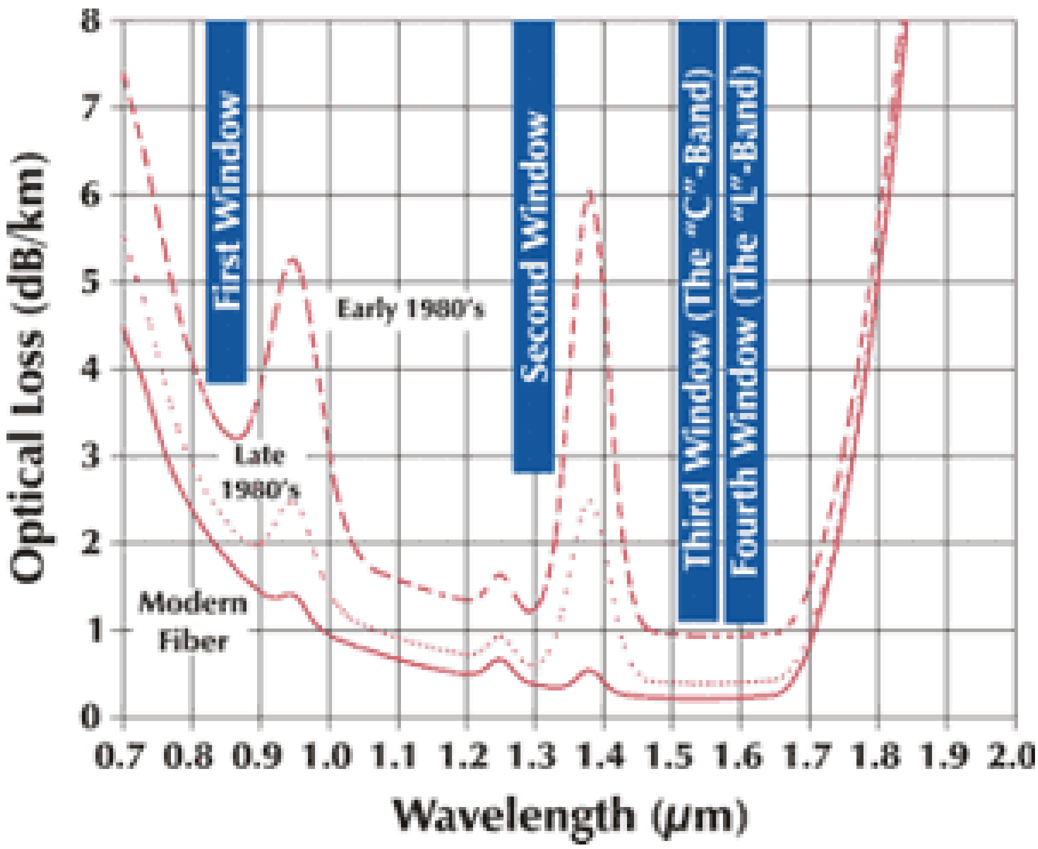

Fig. 4 Variation of fibre attenuation with wavelength.