Çankaya University – ECE Department – ECE 474 (MT)

Student Name :
Student Number :

Date : 05.04.2010
Open book exam, Duration : 2 Hours

Questions

1. (25 Points) The enclosed graph gives the plot of normalized propagation constant b against normalized frequency of fibre V. If a fibre of $V = 5$ is taken, find from this graph, which modes propagate, naming each mode with its labels and also calculating the values of b and β for each mode, given that the fibre and source parameters are $a = 6$ µm, $\lambda = 1.55$ µm, $n_1 = 1.47$, $\Delta = 0.97$ percent.

Solution: Drawing a vertical from $V=5$ on the horizontal axis, we see that modes : $HE_{11} + HE_{21} + TE_{01} + TM_{01} + HE_{12} + HE_{31} + EH_{11}$ are enclosed. So these modes will propagate in the given fibre.

From the given numeric values we can find

$n_2 \simeq n_1(1-\Delta) = 1.4557$

$V = ak(n_1^2 - n_2^2)^{1/2} \simeq 4.9745 \simeq 5$ given in the question

Then from the readings on the vertical axis of the graph;

1) For HE_{11} mode, $b \simeq 0.92$

2) For TE_{01} mode, $b \simeq 0.63$

3) For TM_{01}, HE_{21} modes, $b \approx 0.61$

4) For EH_{11} mode, $b = 0.38$

5) For HE_{31} mode, $b = 0.29$

6) For HE_{12} mode, $b \simeq 0.198$

Using the relation $b = \dfrac{\beta/k - n_2}{n_1 - n_2}$ or $\beta = k[b(n_1 - n_2) + n_2]$

(Eq. 2.2.36 of Agrawal), β for above modes can be found as

1) HE_{11} : $\beta = 5.95 \times 10^6$, 2) TE_{01} : $\beta = 5.94 \times 10^6$

3) TM_{01}, HE_{21} : $\beta \simeq 5.94 \times 10^6$

4) HE_{11} : $\beta = 5.92 \times 10^6$, 5) HE_{31} : $\beta = 5.92 \times 10^6$

6) HE_{12} : $\beta \simeq 5.91 \times 10^6$

All β values are in the range $\underbrace{k_2}_{\frac{2\pi}{\lambda} n_2} < \beta < \underbrace{k_1}_{\frac{2\pi}{\lambda} n_1}$

2. (45 Points) By using the enclosed graph of n and n_g against wavelength and the graph containing the curves of b, $d(Vb)/dV$, $Vd^2(Vb)/d^2V$ against V.
   a. Calculate $D = D_M + D_W$ and comment on your results by comparing it to the given graph of D, D_M, D_W against wavelength,
   b. Calculate dispersion $\Delta T = LD\sigma_\lambda$ and the bandwidth capacity B,

   If the single mode fibre has $a = 3.5$ μm, $\lambda = 1.55$ μm, $n_1 = 1.45$, $\Delta = 0.5$ percent, $L = 15$ km, $\sigma_\lambda = 1$ nm.

Solution: a. If $\lambda = 1.55 \mu m$, $f = 1.9355 \times 10^{14}$ Hz

$\omega = 1.2161 \times 10^{15}$ rad/sec, $V = ak\, n_1 \sqrt{2\Delta} = 2.0572$

From the calculations on the graph, we find

$$\frac{dn_g}{d\lambda} = 5000 \quad \text{and} \quad D_m = \frac{1}{c}\frac{dn_{2g}}{d\lambda} \simeq \frac{1}{c}\frac{dn_g}{d\lambda}$$

$$D_m = \frac{1}{3\times10^8} \times 5000 = 1.67 \times 10^{-5} \text{ per sec per meter per meter}$$

When converted into ps per km per nm

$$D_m = 1.67 \times 10^{-5} \times 10^{12} \times 10^{3} \times 10^{-9} = 16.7 \text{ ps/km/nm}$$

From the given curve we read $D_m = 18$ ps/km/nm

Quite in agreement

$$D_W = -\frac{2\pi\Delta}{\lambda^2}\left[\frac{n_{2g}^2}{n_2\,\omega}\frac{Vd^2(Vb)}{dV^2} + \frac{dn_{2g}}{d\omega}\frac{d(Vb)}{dV}\right]$$

Insert the numeric values and use the conversion

$$\frac{d}{d\omega} = -\frac{\lambda^2}{2\pi c}\frac{d}{d\lambda}, \quad n_2 = n_1(1-\Delta) = 1.4428$$

from $\frac{V d^2(Vb)}{dV^2}$ curve

$$D_W = -\frac{2\pi \times 5\times10^{-3}}{(1.55\times10^{-6})^2}\left[\frac{(1.462)^2}{1.4428\times1.2161\times10^{15}} \times 0.205\right.$$

$$\left. - \frac{(1.55\times10^{-6})^2}{2\pi\times3\times10^8} \times 5\times10^3 \times 0.92\right]$$

↑ From the curve of $\frac{d(Vb)}{dV}$

$\simeq -3.1889\times10^{-6}$ sec per meter per meter

($\times 10^6$ for conversion)

$= -3.1889$ ps/km/nm

From the enclosed graph $D_W \simeq -5$ ps/km/nm

$D = D_m + D_w = 13.511$ ps/km/nm, $D = \sim 14$ ps/km/nm (from the graph)

b. $\Delta T = L D \sigma_\lambda = 15 \times 13.511 \times 1 = 202.67$ ps

$B \simeq \frac{1}{\Delta T} \simeq 4.93$ GHz

3. (30 Points) Answer the following questions as **True** or **False**. For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer.

a) Single mode fibre is obtained by lowering the core radius : True, but this is not the only way to obtain single mode fibre. In general we say that V(related to wavelength, refractive index difference, core radius) must be lower than 2.45

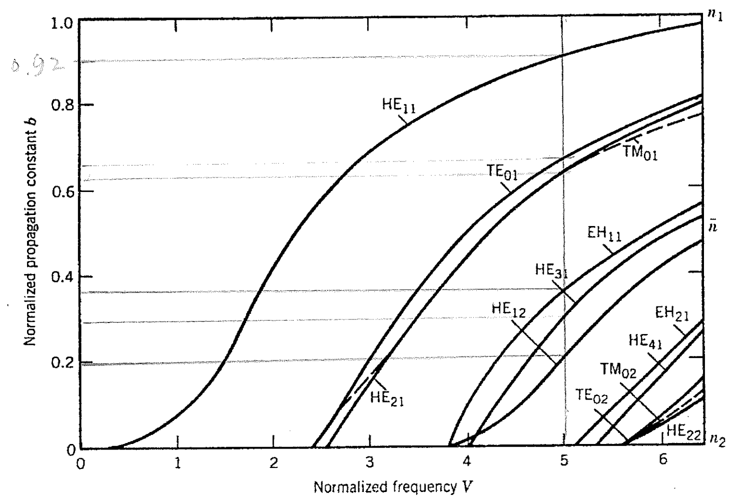
b) Dispersion is the result of refractive index variation with wavelength : True, but this results in material and waveguide dispersions

c) Multimode fibre is obtained by making the refractive index inside the core dependent on radial coordinate : False, this way multimode graded index fibre is obtained

d) In single mode fibre, TE_{01} and TM_{01} modes propagate : False only HE_{11} mode propagates

e) Group velocity expresses the variation of β against angular velocity : False, since it is the inverse and $V_g = \frac{d\omega}{d\beta}$

Figure for Question 1 of ECE 474 MT - 05.04.2010



At $\lambda = 1.55$, From the tangent line $\Delta\lambda = 1.6 - 1 = 0.6\mu m$

$\Delta n_g = 1.4625 - 1.4595 = 3\times10^{-3}$, hence $\frac{dn_g}{d\lambda} = \frac{3\times10^{-3}}{0.6\mu m} = 5000$

Figure for Question 2 of ECE 474 MT - 05.04.2010

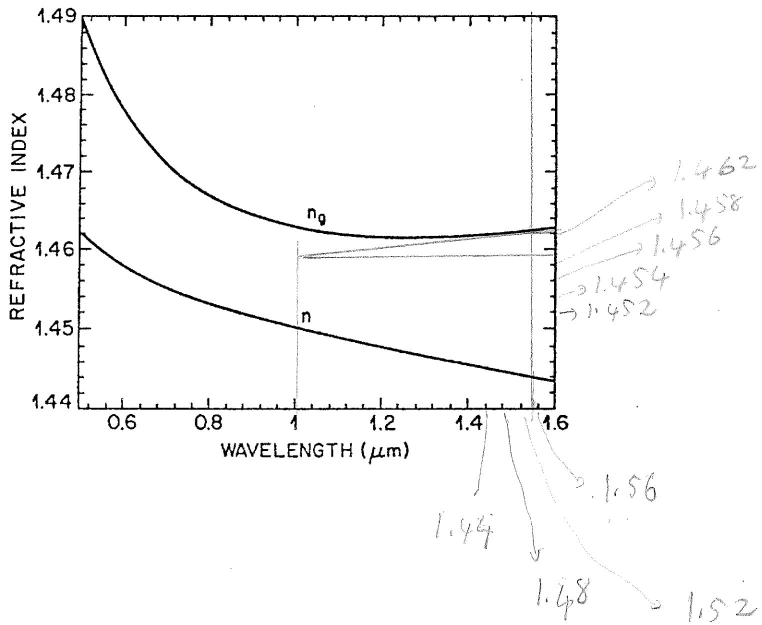


Figure for Question 2 of ECE 474 MT - 05.04.2010

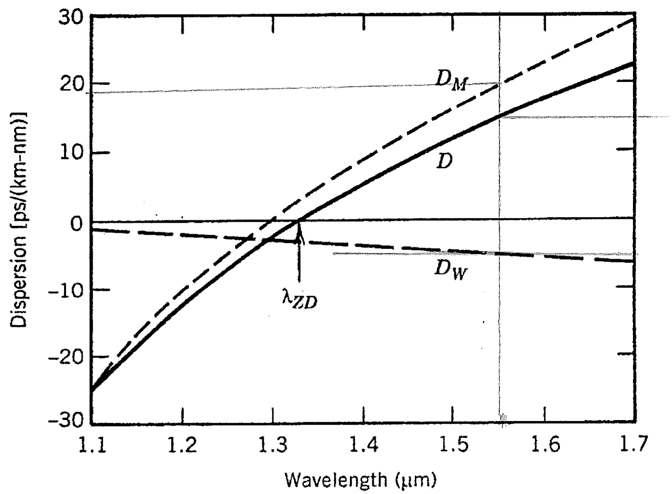


Figure for Question 2 of ECE 474 MT - 05.04.2010