Applied Physics B (2019) 125:152
https://doi.org/10.1007/s00340-019-7269-x

Applied Physics B
Lasers and Optics

Multimode laser beam scintillations in strong atmospheric turbulence

Yahya Baykal[1] · **Hamza Gerçekcioğlu**[2]

Received: 5 February 2019 / Accepted: 22 July 2019 / Published online: 31 July 2019

Abstract

Multimode laser beam scintillation index is formulated at the origin of the receiver plane (on-axis) in strong atmospheric turbulence using the modified Rytov method. Variations of the scintillation index against the turbulence strength and the size of the multimode laser beam are reported for various multimode laser beam contents. It is found that when the multimode content is increased, i.e. when the laser beam contains more modes, and when the multimode content is composed of modes with larger mode numbers, the scintillations tend to reduce in strong atmospheric turbulence. Thus, to achieve smaller intensity fluctuations in atmospheric wireless optics communication systems having long link lengths or operating in strong turbulence, use of multimode laser beams can be recommended.

1 Introduction

Laser beam intensity profiles are known to affect the variation of the received intensity fluctuations in a wireless atmospheric optical communication link. The scintillation index which is a measure for the intensity fluctuations is reported in atmospheric turbulence in many classical publications [1–4]. Influences of various aspects such as the partially coherent beam [5, 6], diffraction size of a transmitting aperture [7], random electromagnetic beams [8, 9], beam types like the Gaussian [10, 11], annular [12] and flat-topped-Gaussian [13, 14] on the scintillations in weak atmospheric turbulence are examined. We have previously reported the effect of beam types in atmospheric turbulence for many different types of optical beams in a review paper [15]. Since in the current paper, we scrutinize the multimode laser beam scintillations, here we specifically mention the work performed for the higher order Gaussian and multimode laser beams. Results for the intensity fluctuations of higher order modes in weak turbulent regime can be traced in the recent publications that cover the anisotropic atmospheric [16], non-Kolmogorov [17], oceanic [18] and marine atmospheric [19] media. Multimode scintillations are also examined in Kolmogorov [20], non-Kolmogorov [21] and underwater turbulence [22, 23].

On the other hand, the behaviour of the scintillations in strong atmospheric turbulent regimes is elaborated in the earlier classical papers in which the theory of saturation [24, 25], comparison of theories for intensity fluctuations [26] and strong fluctuations of fields of optical beams [27] are reported. Later, strong atmospheric turbulence solution for the scintillations is introduced by employing the modified Rytov method [3, 28] which is applied to optical plane [29], annular [30], flat-topped Gaussian [31], higher order mode [32] laser beams and to anisotropic atmospheric turbulence [33]. Scintillations in strong atmospheric turbulence are also evaluated for pulsed Laguerrian beams [34] and in non-Kolmogorov medium [35, 36]. In our recent work, we have investigated the intensity fluctuations in strong oceanic turbulence [37].

In this paper, we report the formulation and evaluations of the scintillation index of multimode laser beams when they are used in strongly turbulent atmospheric medium. It is well known that depending on the type of the media that exhibits different turbulence characteristics, the received intensity fluctuations show very different characteristics. Our earlier results that cover the received intensity fluctuations of multimode laser beams are for the Kolmogorov [20], non-Kolmogorov [21] and underwater turbulence [22] which are all under weak turbulence conditions. The current paper covers the received scintillations of multimode laser beams in strong atmospheric turbulence and it is not possible to

---

✉ Yahya Baykal
y.baykal@cankaya.edu.tr

Hamza Gerçekcioğlu
hgercekcioglu@hotmail.com

[1] Department of Electrical-Electronics Engineering, Çankaya University, Yukarıyurtçu mah., Mimar Sinan cad. No: 4, 06790 Etimesgut, Ankara, Turkey

[2] Ministry of Transport and Infrastructure, General Directorate of Aeronautics and Space Technologies, Gazi Mustafa Kemal Blv. No.128, 06100 Maltepe, Ankara, Turkey

predict the results reported in the current paper from the evaluations reported in our previous works provided in [20–22]. Thus, the results shown in the current paper do not appear in literature. Our motivation is to understand whether the use of multimode laser beam structures will bring advantage in the performance of wireless optical communication systems that operate in long links.

2 Formulation

It is well known that in weak atmospheric turbulence, the scintillation index at the origin of the receiver plane (on-axis) is given by [1]

$$m_{\text{weak}}^2 = 4B_\chi(L), \tag{1}$$

where $B_\chi(L)$ is the log-amplitude correlation function at the origin of the receiver plane which is L distance away from the source plane, i.e. L is the link length.

The multimode laser field at the source plane is [22]

$$u_{\text{MM}}(s_x, s_y) = \sum_{j=1}^{N} A_j H_{n_j}(s_x/\alpha_s) \exp[-(0.5s_x^2/\alpha_s^2)] H_{m_j}(s_y/\alpha_s) \exp[-(0.5s_y^2/\alpha_s^2)], \tag{2}$$

which is obtained by superposing the higher order laser beam structures given by [17]

$$u(s_x, s_y) = H_n(s_x/\alpha_s) H_m(s_y/\alpha_s) \exp\left[-0.5\alpha_s^{-2}(s_x^2 + s_y^2)\right]. \tag{3}$$

Here, n and m are the mode numbers of the higher order laser beams in x and y directions; $H_n(.)$ and $H_m(.)$ are the Hermite polynomials that determine the field distributions in the x and y directions; α_s is the Gaussian source size; s_x, s_y are the x and y components of the transverse source coordinate; A_j is the amplitude of the jth mode and N is the number of higher order beams composing the multimode laser beam structure. In our earlier work [38], using Rytov method, we have reported the formulation of the log-amplitude correlation function for general type optical beams, multimode laser beam being one of them. Then, by introducing the multimode laser beam field given by Eq. (2) into the log-amplitude correlation function formula for general type optical beams [38] and employing Eq. (1), we have formulated the on-axis scintillation index for multimode beams in weak atmospheric turbulence as [21]:

$$m_{\text{weak}}^2 = 4B_\chi(L) = 4\pi \text{Re} \left\{ \int_0^L d\eta \int_0^\infty \kappa d\kappa \int_0^{2\pi} d\theta \left[W_1(\eta, \kappa, \theta, L) + W_2(\eta, \kappa, \theta, L) \right] \Phi_n(\kappa) \right\}, \tag{4}$$

where Re is the real part,

$$W_1(\eta, \kappa, \theta, L) = W_R(\eta, \kappa, \theta, L) W_R(\eta, -\kappa, \theta, L) / X^2(L), \tag{5}$$

$$W_2(\eta, \kappa, \theta, L) = W_R(\eta, \kappa, \theta, L) W_R^*(\eta, \kappa, \theta, L) / |X(L)|^2, \tag{6}$$

$$W_R(\eta, \kappa, \theta, L) = \sum_{\ell=1}^{N} A_\ell i k \alpha_s \varsigma * \exp\left[-\frac{0.5 i \varsigma * (L-\eta)\left(k\alpha_s^2 + i\eta\right)\kappa^2}{k^2 \alpha_s} \right] H_{n_\ell}\left(\frac{\eta - L}{k} \varsigma \kappa \cos\theta \right) \times H_{m_\ell}\left(\frac{\eta - L}{k} \varsigma \kappa \sin\theta \right), \tag{7}$$

$$X(L) = \sum_{\ell=1}^{N} A_\ell \varsigma * \alpha_s H_{n_\ell}(0) H_{m_\ell}(0), \tag{8}$$

$i = (-1)^{0.5}$, $\varsigma = k\alpha_s/\left(k\alpha_s^2 - iL\right)$, $k = 2\pi/\lambda$, λ is the wavelength, * denotes the complex conjugate, η is the distance along the propagation axis, $\kappa\exp(i\theta)$ is the two-dimensional spatial frequency in polar coordinates, κ and θ being the magnitude and the phase of the two-dimensional spatial frequency. In Eq. (4), $\Phi_n(\kappa)$ presents the atmospheric turbulence spectrum. Here, we note that W_1, W_2, W_R and $X(L)$ appearing in Eqs. (5)–(8), respectively, are the propagation terms originating from the Rytov formulation of the log-amplitude correlation function evaluated at the origin of the receiver plane [21]. The scintillation index in Eq. (4) is valid for weak atmospheric turbulence when $\Phi_n(\kappa)$ is the Kolmogorov spectrum. However, in the modified Rytov method in which weak turbulence Rytov solution is extended to cover also the strong atmospheric turbulence solution [28] using the modified turbulence spectrum originally presented for zero inner scale in [28] and rearranged in [32] as:

$$\Phi_{n,e}(\kappa) = \Phi_n(\kappa)\left[\exp\left(-\kappa^2\kappa_x^{-2}\right) + \kappa^{11/3}\left(\kappa^2 + \kappa_y^2\right)^{-11/6}\right], \tag{9}$$

where $\Phi_n(\kappa) = 0.033C_n^2\kappa^{-11/3}$, C_n^2 is the structure constant, κ_x is defined as the large-scale (refractive) or refractive spatial frequency cutoff and κ_y is defined as the small-scale (diffractive) or the diffractive spatial frequency cutoff, which are found to be:

$$\kappa_x = \left\{ \frac{k}{L}\left(\frac{1}{3} - \frac{1}{2}\bar{\Theta} + \frac{1}{5}\bar{\Theta}^2 \right)^{-6/7} \left(\frac{m_{\text{weak}}}{\sigma_R} \right)^{12/7} \left[1+0.56(1+\Theta)\sigma_R^{12/5} \right]^{-1} \right\}^{1/2}, \tag{10}$$

$$\kappa_y = \left\{ \frac{k}{L}\left[3\left(\frac{\sigma_R}{m_{\text{weak}}} \right)^{12/5} + 2.07\sigma_R^{12/5} \right] \right\}^{1/2}, \tag{11}$$

where $\Theta = 1 - Re\left[i\alpha_s L/\left(1 + i\alpha_s L\right)\right]$, $\bar{\Theta} + \Theta = 1$ and $\sigma_R^2 = 1.23 C_n^2 k^{7/6} L^{11/6}$ is the Rytov plane wave scintillation index. To apply amplitude spatial filtering, the large-scale and the small-scale scintillation indices (m_{LS}^2 and m_{SS}^2) are obtained by employing, respectively, the first and the second terms of Eq. (9) in Eq. (4). In Eq. (9), the inner and outer scales are taken as zero and infinity. Employing Eq. (9) in Eq. (2), integrating over θ and κ, the small-scale and the large-scale scintillation indices are obtained. Then the combined scintillation index can be expressed for strong atmospheric turbulence as [4]:

$$m^2 = \exp\left(m_{\mathrm{LS}}^2 + m_{\mathrm{SS}}^2\right) - 1, \tag{12}$$

where m_{LS}^2 and m_{SS}^2 are found to be

$$m_{\mathrm{LS}}^2 = 0.49\left(\frac{1}{3} - \frac{1}{2}\bar{\Theta} + \frac{1}{5}\bar{\Theta}^2\right)\sigma_R^2 \eta_x^{7/6}, \tag{13}$$

$$m_{\mathrm{SS}}^2 = 1.27\sigma_R^2 \eta_Y^{-5/6}. \tag{14}$$

Here, $\eta_x = \frac{L\kappa_x^2}{k}$, $\eta_y = \frac{L\kappa_y^2}{k}$ and Re denotes the real part. Using Eqs. (13) and (14) in Eq. (12), the on-axis scintillation index for multimode laser beam in strong atmospheric turbulence is obtained. We note that Eq. (10) is valid in horizontal links and for multimode laser beams composed of modes having even mode numbers. The results obtained by evaluating Eq. (10) are presented in Sect. 3.

3 Results and discussion

In all the figures, A_j is taken to be unity for all j and the source sizes of the beams composing the multimode laser beam are taken to be the same. The wavelength is taken to be 1.55 μm in all the figures. The horizontal axes of Figs. 1, 2 and 5 are $\sigma_R = \left(1.23 C_n^2 k^{7/6} L^{11/6}\right)^{1/2}$ which is known [4] to be the parameter to determine the strength of atmospheric turbulence. For $\sigma_R^2 < 1$, atmospheric turbulence is in the weak fluctuations regime and when $\sigma_R^2 > 1$, strong turbulence regime dictates. For the interim values of σ_R^2, atmospheric turbulence is said to have moderate strength. Modified Rytov method solution is known to cover all of the weak, moderate and strong atmospheric turbulence regimes [3, 28]. Thus, the scintillation index expression given by Eq. (10) actually covers all the atmospheric turbulence regimes.

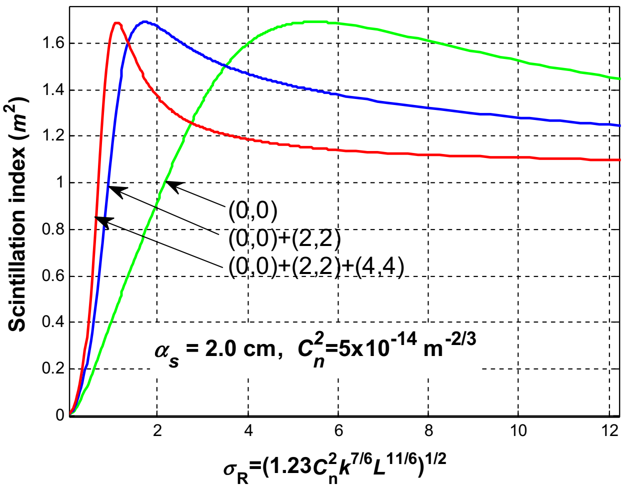

Fig. 1 Scintillation index in strong atmospheric turbulence versus σ_R for single mode (0,0), 2 modes (0,0), (2,2) and 3 modes (0,0), (2,2), (4,4)

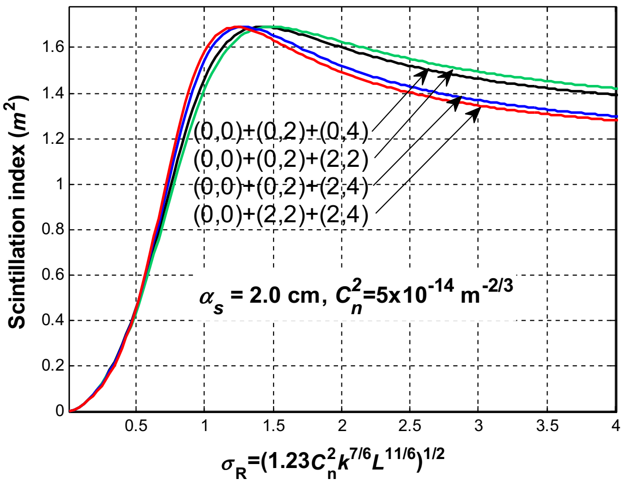

Fig. 2 Scintillation index in strong atmospheric turbulence versus σ_R for various 3 mode combinations

In Fig. 1, the scintillation index in strong atmospheric turbulence is plotted versus the plane wave scintillation index for various multimode laser beam contents where the single mode presents the fundamental mode (0,0), i.e. the Gaussian beam, and the other curves present the mode combinations of (0,0) + (2,2) for the 2 modes and (0,0) + (2,2) + (4,4) for the 3 modes. It is observed from Fig. 1 that as the turbulence strength increases, being valid for all the chosen mode content combinations, the scintillation index of multimode laser beams first increases, after reaching a maximum value, starts to decrease and is retained at the saturation value. Examining the scintillation index in Fig. 1 at the same but large turbulence strength values, it is seen that as the number of beams in the multimode beam increases, the scintillation index becomes smaller. In Fig. 2, the scintillation index in strong atmospheric turbulence is examined against the turbulence strength for multimode laser beams where each curve is composed of 3 modes with different contents. It is seen from Fig. 2 that whatever the contents of the 3 modes composing the multimode laser beam is, increase in the turbulence strength first increases the scintillation index of multimode laser beams in strong atmospheric turbulence, then causes the scintillations to reach certain values and eventually the scintillation index stays at a certain saturation value for each curve as the atmospheric turbulence becomes stronger. Looking at the curves of Fig. 2 at the fixed the turbulence strength values, it is not easy to reach a general trend, however, comparison of the upper two curves with the lower two curves indicates that the 3 mode multimode beams whose contents have higher mode numbers exhibit lower intensity fluctuations in strong atmospheric turbulence. Figure 3 shows the variation of the scintillation index in strong atmospheric turbulence of multimode beams having different mode contents, versus their source size. σ_R in Fig. 3 is taken to be a fixed value of 2.73 which falls in the strong atmospheric turbulence regime. It is seen from Fig. 3 that irrespective of the number of beams composing the multimode laser beam, as the source size increases, the intensity of multimode beams first fluctuates less, reaches a minimum value, then starts to increase and attains a saturation value after a slight decrease. That is, in strong atmospheric turbulence, larger sized multimode sources eventually reach a steady value at sufficiently large source sizes. At the fixed source size, it is observed from Fig. 3 that the intensity fluctuations of multimode laser beams in sufficiently strong atmospheric turbulence are lowered when the number of beams composing the multimode content is larger. Scintillation index in strong atmospheric turbulence versus the source size of the multimode laser beam where each curve is composed of 3 modes with different mode contents is presented in Fig. 4. Observation from Fig. 4 is that for all the multimode laser beams, each containing the same number of beams with different mode numbers, as the source size increases, the scintillation index in strong atmospheric turbulence first decreases, makes a minimum, then starts to increase and reaches a steady state saturation value. In Fig. 4, similar to Fig. 3, the plane wave scintillation index is again kept at the same value of 2.73, i.e. strong atmospheric condition is retained. Investigating the scintillation index in Fig. 4 at the fixed source size, comparison of the upper two curves with the lower two curves yields, as in Fig. 2, that the 3 mode multimode beams whose contents have higher mode numbers exhibit smaller scintillation index in strong atmospheric turbulence. Figures 1 and 2 are for fixed source size and varying atmospheric turbulence strengths, and Figs. 3 and 4 are for fixed atmospheric turbulence strength and varying source sizes. It is also of interest how the scintillation index of multimode laser beam is effected with the source size at varying atmospheric turbulence strengths. For this purpose, Fig. 5 is provided in which the scintillation index in strong atmospheric turbulence versus the turbulence strength of one multimode laser beam composed of the modes (0,0) + (2,2) + (2,4) is plotted for different source sizes. The observation from Fig. 5 is that the trend seen in Fig. 1 holds to be true for all the source sizes of the multimode beam, i.e. as the turbulence strength increases, the scintillation index of the multimode laser beam first increases and reaches certain value, then this value is retained as the atmospheric turbulence becomes stronger. When the turbulence strength increases to a large extent, it is seen that the multimode beam with the larger source size attains slightly smaller scintillations but at such large turbulence strengths, the scintillations of all the multimode beams of different source sizes merge to a certain saturation value.

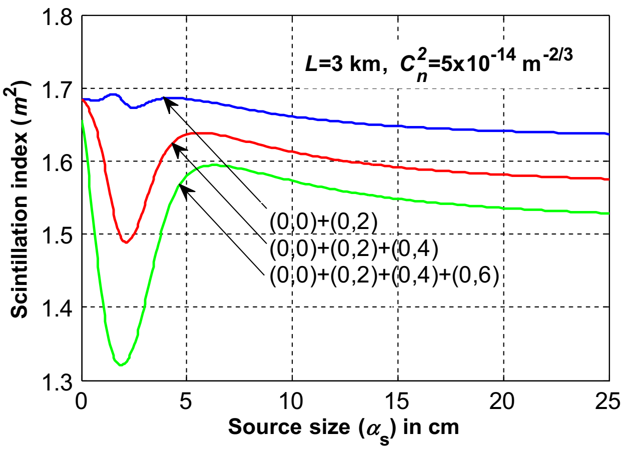

Fig. 3 Scintillation index in strong atmospheric turbulence versus source size for single mode (0,0), 2 modes (0,0), (2,2) and 3 modes (0,0), (2,2), (4,4)

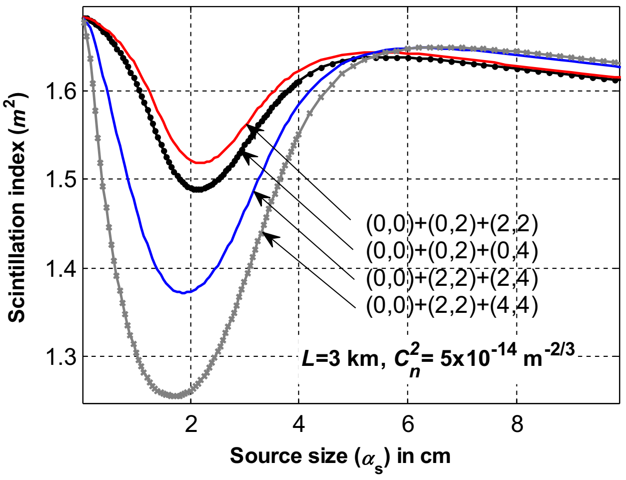

Fig. 4 Scintillation index in strong atmospheric turbulence versus source size for various 3 mode combinations

The results presented in this section cover the scintillation index for multimode laser beams evaluated at the origin of the receiver plane (on-axis) where the multimode contents are composed of even higher order laser beams whose mode numbers in *x* and *y* directions, i.e. *n* and *m*, are all even.

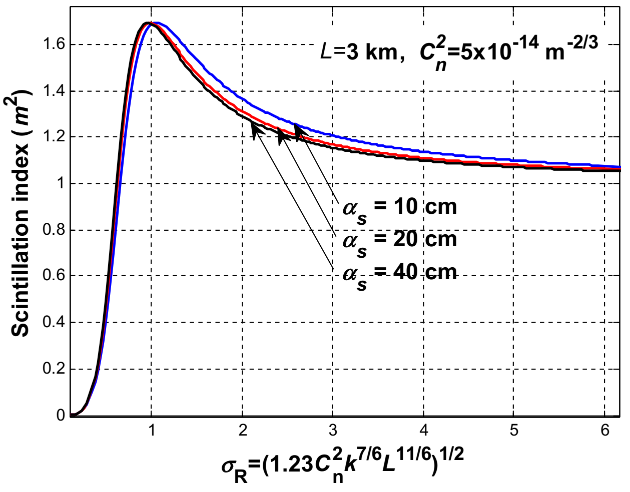

Fig. 5 Scintillation index in strong atmospheric turbulence versus σ_R for different source sizes

Multimode beams composed of all even higher order modes have the property that the point of maximum of the average intensity distribution of the corresponding multimode beam structure is at the origin of the receiver plane, and we calculate the scintillation index at the origin of the receiver plane (on-axis). Thus, the on-axis scintillation index is in fact calculated as a function of σ_R at the point of maximum of the average intensity distribution of the corresponding multimode beam structure. In this way, the results are brought to one type for correct comparison.

4 Conclusion

The intensity fluctuations quantified by the scintillation index are evaluated for laser beams of multimode content in strongly turbulent atmospheric medium. When the strength of atmospheric turbulence increases, before reaching a maximum value, the scintillation index of multimode laser beam is found first to increase, then to decrease and to stay at the saturation value. At large turbulence strength values, when the number of beams in the multimode content becomes larger, the scintillation index tends to become smaller. Comparing the scintillation index of multimode beams having the same number of beams in strong atmospheric turbulence, it is found that the multimode structures possessing beams with larger mode numbers attain smaller scintillation values. The scintillation index in strong atmospheric turbulence is found to decrease when multimode beams having larger number of beams are employed.

Based on the results found in this paper, in horizontal wireless optical communication links having very long link lengths, use of multimode laser beams with large number of beam contents can be recommended to reduce the scintillation noise, thus to improve the performance of such links. It is noted that the behaviour of the scintillation index of multimode laser beams can show different characteristics when they are used as excitations in other types of turbulent media as reported in [20–22] for weakly turbulent Kolmogorov, non-Kolmogorov atmospheric and underwater turbulence, respectively.

References

1. V.I. Tatarskii, *Wave Propagation in a Turbulent Medium* (McGraw-Hill, New York, 1961)
2. A. Ishimaru, *Wave Propagation and Scattering in Random Media*, vol. 2 (Academic, Cambridge, 1978)
3. L.C. Andrews, R.L. Phillips, C.Y. Hopen, *Laser Beam Scintillation with Applications* (SPIE, Washington, 2001)
4. L.C. Andrews, R.L. Phillips, *Laser Beam Propagation Through Random Media* (SPIE, Washington, 2005)
5. A.I. Kon, V.I. Tatarskii, On the theory of the propagation of partially coherent light beams in a turbulent atmosphere. Radiophys. Quantum Electron. **15**, 1187–1192 (1972)
6. O. Korotkova, L.C. Andrews, R.L. Phillips, Model for a partially coherent Gaussian beam in atmospheric turbulence with application in lasercom. Opt. Eng. **43**, 330–341 (2004)
7. V.A. Banakh, V.L. Mironov, Influence of the diffraction size of the transmitting aperture and of the turbulence spectrum on the intensity fluctuations of laser radiation. Sov. J. Quantum Electron. **8**, 875–878 (1978)
8. O. Korotkova, Control of the intensity fluctuations of random electromagnetic beams on propagation in weak atmospheric turbulence. Proc. SPIE **6105**, 61050V (2006)
9. O. Korotkova, Changes in the intensity fluctuations of a class of random electromagnetic beams on propagation. J. Opt. A **8**, 30–37 (2006)
10. J.C. Leader, Beam-intensity fluctuations in atmospheric turbulence. J. Opt. Soc. Am. **71**, 542–558 (1981)
11. L.C. Andrews, M.A. Al-Habash, C.Y. Hopen, R.L. Phillips, Theory of optical scintillation: Gaussian-beam wave model. Waves Random Media **11**, 271–291 (2001)
12. F.E.S. Vetelino, L.C. Andrews, Annular Gaussian beams in turbulent media. Proc. SPIE **5160**, 86–97 (2004)
13. Y. Baykal, H.T. Eyyuboğlu, Scintillation index of flat-topped Gaussian beams. Appl. Opt. **45**, 3793–3797 (2006)
14. D.C. Cowan, L.C. Andrews, Effects of atmospheric turbulence on the scintillation and fade probability of flattened Gaussian beams. Opt. Eng. **47**, 026001 (2008)
15. Y. Baykal, H.T. Eyyuboğlu, Y. Cai, Effect of beam types on the scintillations: a review. Proc. SPIE **7200**, 720002 (2009)
16. Y. Baykal, Y. Luo, X. Ji, Scintillations of higher order laser beams in anisotropic atmospheric turbulence. Appl. Opt. **55**, 9422–9426 (2016)
17. Y. Baykal, Scintillations of higher-order laser beams in non-Kolmogorov medium. Opt. Lett. **39**, 2160–2163 (2014)
18. Y. Baykal, Higher order mode laser beam scintillations in oceanic medium. Waves Random Complex Media **26**, 21–29 (2015)
19. Y. Baykal, Higher-order laser beam scintillation in weakly turbulent marine atmospheric medium. J. Opt. Soc. Am. A **33**, 758–763 (2016)
20. Y. Baykal, Scintillation index for a multimode laser incidence in weak atmospheric turbulence. Opt. Commun. **62**, 295–299 (1987)

21. Y. Baykal, Multimode laser beam scintillations in non-Kolmogorov turbulence. IEEE J. Sel. Area Commun. **33**, 1883–1889 (2015)
22. Y. Baykal, Intensity fluctuations of multimode laser beams in underwater medium. J. Opt. Soc. Am. A **32**, 593–598 (2015)
23. S.A. Arpali, Y. Baykal, Ç. Arpali, BER evaluations for multimode beams in underwater turbulence. J. Mod. Optic **63**, 1297–1300 (2016)
24. K.S. Gochelashvili, V.G. Pevgov, V.I. Shishov, Saturation of fluctuations of the intensity of laser radiation at large distances in a turbulent atmosphere (Fraunhofer zone of transmitter). Sov. J. Quantum Electron. **4**, 632–637 (1974)
25. R.J. Hill, Theory of saturation of optical scintillation by strong turbulence-plane-wave variance and covariance and spherical-wave covariance. J. Opt. Soc. Am. **72**, 212–222 (1982)
26. R.L. Fante, Comparison of theories for intensity fluctuations in strong turbulence. Radio Sci. **11**, 215–220 (1976)
27. S.I. Belousov, I.G. Yakushkin, Strong fluctuations of fields of optical beams in randomly inhomogeneous media. Sov. J. Quantum Electron. **10**, 301–304 (1980)
28. L.C. Andrews, R.L. Phillips, C.Y. Hopen, M.A. Al-Habash, Theory of optical scintillation. J. Opt. Soc. Am. A **16**, 1417–1429 (1999)
29. I. Toselli, L.C. Andrews, R.L. Phillips, V. Ferrero, Scintillation index of optical plane wave propagating through non-Kolmogorov moderate-strong turbulence. Proc. SPIE **6747**, 67470B (2007)
30. H. Gerçekcioğlu, Y. Baykal, C. Nakiboğlu, Annular beam scintillations in strong turbulence. J. Opt. Soc. Am. A **27**, 1834–1839 (2010)
31. H. Gerçekcioğlu, Y. Baykal, Scintillation index of flat-topped Gaussian laser beam in strongly turbulent medium. J. Opt. Soc. Am. A **28**, 1540–1544 (2011)
32. Y. Baykal, Scintillation index of higher order mode laser beams in strong turbulence. Opt. Commun. **386**, 53–56 (2017)
33. L. Cui, B. Xue, F. Zhou, Modified anisotropic turbulence refractive-index fluctuations spectral model and its application in moderate-to-strong anisotropic turbulence. J. Opt. Soc. Am. A **33**, 483–491 (2016)
34. V.A. Banakh, L.O. Gerasimova, Strong scintillations of pulsed Laguerrian beams in a turbulent atmosphere. Opt. Express **24**, 19264–19277 (2016)
35. P. Deng, X.-H. Yuan, D. Huang, Scintillation of a laser beam propagation through non-Kolmogorov strong turbulence. Opt. Commun. **285**, 880–887 (2012)
36. X. Yi, Z. Liu, P. Yue, Optical scintillations and fade statistics for FSO communications through moderate-to-strong non-Kolmogorov turbulence. Opt. Laser Technol. **47**, 199–207 (2013)
37. Y. Baykal, Scintillation index in strong oceanic turbulence. Opt. Commun. **375**, 15–18 (2016)
38. Y. Baykal, Formulation of correlations for general-type beams in atmospheric turbulence. J. Opt. Soc. Am. A **23**, 889–893 (2006)