**ECE 474 / PWE TEST**

**MATLAB CODE**

clear all;clc ;close all;warning off all

syms A0 Ac Fi0 Fi alfas Fs alfa r k z kz

%%%% Field of a plane wave after propagating a distance of z

UR = A0\*exp(j\*Fi0 + j\*k\*z);

UR = exp(j\*k\*z + j\*k\*r^2/(2\*z))/(4\*pi\*z);

UR = sin(Fi/2)\*sin(alfa\*r)/sqrt(alfa\*r)\*exp(-j\*kz\*z);k2 = alfa^2 + kz^2;

%%% Derivatives of U(R) wrt r Fi and z

URr = diff(UR,'r');URrr = diff(URr,'r');URFi = diff(UR,'Fi');URFiFi = diff(URFi,'Fi');URz = diff(UR,'z');URzz = diff(URz,'z');

% %%% Alternative expressions for derivatives

%URr = diff(UR,'r');URrr = diff(UR,2,'r');URFiFi = diff(UR,2,'Fi');URz = diff(UR,'z');URzz = diff(UR,2,'z');

% %%% Analytic expression of Helmholtz equation

%HE = URrr + URr/r + URFiFi/r^2 + 1\*URzz + k^2\*UR;simplify(HE)

HE = r^2\*URrr + r\*URr + URFiFi + r^2\*URzz + r^2\*k2\*UR;simplify(HE)

%simplify(r^2\*URrr + r\*URr)

% % %%% Numeric evaluation

% lamda = 1.55e-6;k = 2\*pi/lamda;r = 1e-2;alfa = 10;Fi = pi/2.3;z = 1e3;

% HE = eval(URrr + URr/r + URFiFi/r^2 + 1\*URzz + k^2\*UR)

% %%% The case of a spherical wave, since spherical wave representation is

% %%% an approximation, it does not satisfy Helmholtz equation

% UR = exp(j\*k\*z + j\*k\*r^2/(2\*z))/(4\*pi\*z);

% URr = diff(UR,'r');URrr = diff(UR,2,'r');URFiFi = diff(UR,2,'Fi');URzz = diff(UR,2,'z');;

% % %%% Analytic expression

%HE = URrr + URr/r + URFiFi/r^2 + URzz + k^2\*UR;simplify(HE)

%

% %%% The case of a Gaussian beam wave, this does not satisfy Helmholtz equation, but PWE

% alfa = 1/(k\*alfas^2) + 0.5\*j/Fs;

UR = Ac\*exp(-k\*alfa\*r^2/(1 + 2\*j\*alfa\*z))/(1 + 2\*j\*alfa\*z);%UR = Ac\*exp(-k\*alfa\*r^2);

URr = diff(UR,'r');URrr = diff(UR,2,'r');URFiFi = diff(UR,2,'Fi');URz = diff(UR,'z');

% % %% Analytic expression

PWE = 1.0\*URrr + URr/r + URFiFi/r^2 + 2\*j\*k\*URz; %['PWE result of Gaussian beam = ']

simplify(PWE)