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# Optimization of spectrally flat and broadband single-pump fiber optic parametric amplifiers 

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#### Abstract

Using well-known theory for fiber optical parametric amplifiers in the no-depletion approximation, a simple condition relating the second- and fourth-order dispersion parameters $\beta_{2}$ and $\beta_{4}$ is demonstrated which ensures ripple-free, broadband gain spectra using realistic fiber parameters. Inclusion of pump depletion gives rise to expressions which enable optimization of fiber length and the maximum achievable ripple-free gain.


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## 1. Introduction

Fiber optical parametric amplifiers (FOPAs) have been intensively studied in recent years due to their potential use for amplification and wavelength conversion in multi-terabit/s dense wavelength division multiplexing (DWDM) transmission systems [1-13]. FOPAs have the advantage of being able to operate in any of the telecom bands (S-C-L) depending upon pump wavelength and the fiber zerodispersion wavelength, which, in principle, can be appropriately tailored.

[^1]In order to be a practical amplifier in a DWDM system, the FOPA should exhibit high gain, large bandwidth and should be spectrally flat, among other requirements. Researchers have recently demonstrated a broadband FOPA with 49 dB gain using a multi-segment fiber design [2], and a very broadband ( 200 nm , considering both the signal and idler bands) Raman-assisted FOPA [6]. Both suffered from severely non-flat gain spectra, as do most FOPAs. In order to flatten the gain without the use of filters, researchers have recently investigated both multi-segment $[4,13,14]$ and dualpump designs $[3,5,11,15,16]$.

In this paper, we revisit the simple single-pump, single-fiber optical parametric amplifier previously studied by several authors [1,7,17-20]. In particular, we further exploit the work of Marhic et al.,
where it was shown theoretically that gain spectra can be flattened and total bandwidth can be broadened by taking into account the fiber secondand fourth-order dispersion parameters $\beta_{2}$ and $\beta_{4}$ [1]. Here we concentrate our attention on the conditions required for obtaining a very flat, rip-ple-free gain band, thus ensuring gain uniformity over a large portion of the parametric gain spectrum. Design rules for achieving such broadband, flat parametric gain spectra are presented which are valid for any realistic set of fiber parameters, allowing the design of single-pump, single-fiber FOPAs with $3-\mathrm{dB}$ bandwidths of over 40 nm (single-sided) and gain greater than 50 dB , which is ripple-free across $\sim 30 \%$ of the total spectrum without the use of any gain flattening devices. The key to obtaining these results is the proper exploitation of the fiber second- and fourth-order dispersion parameters $\beta_{2}$ and $\beta_{4}$, respectively. After completion of this work, the article of [15] came to our attention, where some of their results agree entirely with some of the results found independently and reported here.

## 2. Theory

In the no-depletion approximation the parametric amplification is described by the signal power gain $[1,18,21]$
$G_{\mathrm{s}}(L)=\frac{P_{\mathrm{s}}(L)}{P_{\mathrm{s}}(0)}=1+\left[\frac{\gamma P_{\mathrm{p}}}{g} \sinh (g L)\right]^{2}$,
where $P_{\mathrm{p}}$ and $P_{\mathrm{s}}$ are the pump and signal powers in the fiber, $\gamma$ is the fiber nonlinear coefficient ( $\gamma=2 \pi n_{2} / \lambda A_{\text {eff }}$ ), $L$ is the fiber length and $g$ is the parametric gain parameter given by $[1,21]$
$g^{2}=\left(\gamma P_{\mathrm{p}}\right)^{2}-\kappa^{2} / 4=-\Delta \beta\left(\Delta \beta / 4+\gamma P_{\mathrm{p}}\right)$,
where $\kappa=\Delta \beta+2 \gamma P_{\mathrm{p}}$ and $\Delta \beta=\beta\left(\omega_{\mathrm{s}}\right)+\beta\left(\omega_{\mathrm{i}}\right)-$ $2 \beta\left(\omega_{\mathrm{p}}\right)$. Expanding the terms $\beta\left(\omega_{\mathrm{s}}\right)$ and $\beta\left(\omega_{\mathrm{i}}\right)$ around $\omega_{\mathrm{p}}$ and retaining terms up to fourth-order yields
$\Delta \beta=\beta_{2} \Omega^{2}+\frac{\beta_{4}}{12} \Omega^{4}$,
where $\Omega=\omega_{\mathrm{s}}-\omega_{\mathrm{p}}$ is the frequency detuning of the signal from the pump and $\beta_{m}=d^{m} \beta /\left.d \omega^{m}\right|_{\omega=\omega_{\mathrm{p}}}$.

Note that the odd terms drop out of Eq. (3) since $\omega_{\mathrm{i}}-\omega_{\mathrm{p}}=-\left(\omega_{\mathrm{s}}-\omega_{\mathrm{p}}\right)$.

In order to have maximum parametric gain, the nonlinear phase mismatch $\kappa$ should be equal to zero, i.e., the interaction should be phasematched. To illustrate this we show in Figs. 1(a) and (b) the behavior of the nonlinear phase mismatch and the corresponding fiber gain as a function of the signal detuning for a pump at 1550 nm . Note that we have shown only one side of the curves, which are approximately symmetric about the pump wavelength. To obtain these figures, we used $\gamma P_{\mathrm{p}}=15 \mathrm{~km}^{-1}$, which could be easily achieved with a CW pump in a highly nonlinear fiber, $L=0.20 \mathrm{~km}, \beta_{4}=2.5 \times 10^{-4} \mathrm{ps}^{4} \mathrm{~km}^{-1}$, an experimental value found in the literature [ 1,21 ], and we varied $\beta_{2}$ from -0.030 to $-0.065 \mathrm{ps}^{2} / \mathrm{km}$, typical values which one would obtain when pumping very near the zero-dispersion wavelength $\lambda_{0}$. For $\beta_{2}=-0.030 \mathrm{ps}^{2} / \mathrm{km}$ the interaction is never phasematched, however the gain spectrum has a maximum at the minimum of $\kappa$. On the other hand, when $\beta_{2}=-0.065 \mathrm{ps}^{2} / \mathrm{km}$, the interaction is phasematched for two different signal detunings, which leads to two peaks of maximum gain on each side of the gain spectrum. The depression in the gain spectrum between the two peaks is due to the non-zero phase mismatch in this region. Finally, when the minimum of $\kappa$ coincides with $\kappa=0$ (which occurs for $\beta_{2}=-0.050 \mathrm{ps}^{2} / \mathrm{km}$ in this example), we have a condition of minimum sensitivity to phase mismatch, and the corresponding gain spectrum exhibits a broadened, flattened band [15].

It is thus possible to find a relationship between $\beta_{2}$ and $\beta_{4}$, which results in a broadband, flat gain spectrum by requiring the minimum of $\kappa$ (occurring at $\Omega^{2}=-6 \beta_{2} / \beta_{4}$ ) to coincide with the phasematch condition $\kappa=0$. The resulting relationship, which is presented here for the first time in a simple form, is
$\beta_{2}=-\left(\frac{2}{3} \gamma P_{\mathrm{p}} \beta_{4}\right)^{1 / 2}$.
This relation shows the usual requirement of $\beta_{2}<0$ for broadband FOPAs, while also requiring $\beta_{4}$ to be positive [1]. The $\beta_{4}$ is assumed to be approximately constant over a large range of signal


Fig. 1. Nonlinear phase mismatch (a) and gain spectra (b) obtained using $\beta_{4}=2.5 \times 10^{-4} \mathrm{ps}^{4} \mathrm{~km}^{-1}$, with a fixed fiber nonlinearity $\gamma P_{\mathrm{p}}=15 \mathrm{~km}^{-1}$ and a fiber length $L=0.20 \mathrm{~km}$.
wavelengths near $\lambda_{0}$, while $\beta_{2}$ depends on the dispersion slope of the fiber and the detuning of the pump wavelength from $\lambda_{0}$. Given the dispersion values for wavelengths near $\lambda_{0}$, it is not difficult to determine the $\beta_{4}$ of the fiber [6,21]. Subsequently, Eq. (4) yields the $\beta_{2}$ and thus the pump wavelength required to achieve a flat gain spectrum for a given fiber.

## 3. Results of numerical simulations

In what follows, we demonstrate how the use of Eq. (4) results in flat, broadband gain spectra for any realistic set of experimental FOPA parameters. In all the figures which follow, we have constrained $\beta_{2}$ to the value given by Eq. (4) for the parameters used. While exhibiting behavior generally identical to gain calculations which neglect $\beta_{4}$, the important feature is that the spectra remain flat, indicating the generality of Eq. (4). Note that in Figs. 2-5 we show only the long-wavelength side of the total gain spectra for a pump wavelength of 1550 nm . In Fig. 2 we show several flat gain spectra which one obtains by varying $\beta_{4}$ from $1.0 \times 10^{-4} \mathrm{ps}^{4} \mathrm{~km}^{-1}$ to $5.0 \times 10^{-3} \mathrm{ps}^{4} \mathrm{~km}^{-1}$, while fixing $\gamma P_{p}=15 \mathrm{~km}^{-1}$ and $L=0.20 \mathrm{~km}$. We see that bandwidth increases significantly as $\beta_{4}$ is reduced, as expected since $\left|\beta_{2}\right|$ is simultaneously reduced, while spectral flatness is continually preserved over a large range of signal wavelengths.

In Fig. 3, we vary $\gamma P_{\mathrm{p}}$ from 2 to $20 \mathrm{~km}^{-1}$, while fixing the fiber length at $L=0.20 \mathrm{~km}$ and $\beta_{4}=2.5 \times 10^{-4} \mathrm{ps}^{4} \mathrm{~km}^{-1}$. Again flatness is preserved, while the total gain bandwidth and the maximum gain increase as $\gamma P_{p}$ increases, as expected. The $3-\mathrm{dB}$ bandwidth remains approximately constant $(\sim 40 \mathrm{~nm})$ for $\gamma P_{\mathrm{p}}$ values which leads to useful, i.e., $>10 \mathrm{~dB}$, gain spectra.

In Fig. 4 the fiber length is changed while maintaining $\beta_{4}=2.5 \times 10^{-4} \mathrm{ps}^{4} \mathrm{~km}^{-1}$ and $\gamma P_{\mathrm{p}}=$ $15 \mathrm{~km}^{-1}$. We clearly see that the maximum flat


Fig. 2. Flat gain spectra obtained using values of $\beta_{2}$ given by Eq. (4) while varying $\beta_{4}$ from $1.0 \times 10^{-4} \mathrm{ps}^{4} \mathrm{~km}^{-1}$ to $5.0 \times$ $10^{-3} \mathrm{ps}^{4} \mathrm{~km}^{-1}$, with a fixed fiber nonlinearity $\gamma P_{\mathrm{p}}=15 \mathrm{~km}^{-1}$ and a fiber length $L=0.20 \mathrm{~km}$.


Fig. 3. Flat gain spectra obtained using values of $\beta_{2}$ given by Eq. (4) by varying $\gamma P_{\mathrm{p}}$, while fixing $\beta_{4}=2.5 \times 10^{-4} \mathrm{ps}^{4} \mathrm{~km}^{-1}$ and $L=0.20 \mathrm{~km}$.


Fig. 4. Flat gain spectra obtained using values of $\beta_{2}$ given by Eq. (4) for various fiber lengths. Here $\beta_{4}=2.5 \times 10^{-4} \mathrm{ps}^{4} \mathrm{~km}^{-1}$ and $\gamma P_{\mathrm{p}}=15 \mathrm{~km}^{-1}$.
gain increases approximately linearly with $L$, as expected. The total gain bandwidth remains almost constant, whereas the $3-\mathrm{dB}$ gain bandwidth diminishes slightly for long fiber lengths.

In Fig. 4 we see up to 60 dB gain for $L=0.5$ km . Experimental values of $\sim 50 \mathrm{~dB}$ have already been demonstrated [2,7] and very recently, 60 dB gain has been experimentally demonstrated [22]. Note from Eq. (1) that the no-depletion approximation leads to unlimited growth of the gain as a function of the FOPA parameters, for example, the fiber length. Also, in this approximation the
gain spectrum is independent of the input signal power. In practice, however, pump depletion and signal levels limit the maximum gain achievable [7,23], so these must be included if we want a realistic FOPA design.

Pump depletion is governed by the coupled equations, Eq. (5), that describe the evolution of the pump, signal, and idler along the fiber [7,17-19]

$$
\begin{align*}
\frac{\mathrm{d} P_{\mathrm{p}}}{\mathrm{~d} z} & =-4 \gamma\left(P_{\mathrm{p}}^{2} P_{\mathrm{s}} P_{\mathrm{i}}\right)^{1 / 2} \sin \theta \\
\frac{\mathrm{~d} P_{\mathrm{s}}}{\mathrm{~d} z} & =2 \gamma\left(P_{\mathrm{p}}^{2} P_{\mathrm{s}} P_{\mathrm{i}}\right)^{1 / 2} \sin \theta, \\
\frac{\mathrm{~d} P_{\mathrm{i}}}{\mathrm{~d} z} & =2 \gamma\left(P_{\mathrm{p}}^{2} P_{\mathrm{s}} P_{\mathrm{i}}\right)^{1 / 2} \sin \theta,  \tag{5}\\
\frac{\mathrm{~d} \theta}{\mathrm{~d} z}= & \Delta \beta+\gamma\left\{2 P_{\mathrm{p}}-P_{\mathrm{s}}-P_{\mathrm{i}}+\left[\left(P_{\mathrm{p}}^{2} P_{\mathrm{i}} / P_{\mathrm{s}}\right)^{1 / 2}\right.\right. \\
& \left.\left.+\left(P_{\mathrm{p}}^{2} P_{\mathrm{s}} / P_{\mathrm{i}}\right)^{1 / 2}-4\left(P_{\mathrm{s}} P_{\mathrm{i}}\right)^{1 / 2}\right] \cos \theta\right\},
\end{align*}
$$

$P_{\mathrm{p}}, P_{\mathrm{s}}$, and $P_{i}$ are the pump, signal and idler powers in the fiber and $\theta$ is a relative phase factor between the fields, defined as $\theta(z)=\Delta \beta z+\phi_{\mathrm{s}}(z)+$ $\phi_{\mathrm{i}}(z)-2 \phi_{\mathrm{p}}(z)$, where $\phi_{\mathrm{s}}, \phi_{\mathrm{i}}$ and $\phi_{\mathrm{p}}$ are the phases of the respective fields. The set of Eqs. (5) can be analytically solved for the case where fiber loss is neglected $[19,24,25]$ as briefly explained in Appendix A.

In Fig. 5 we show the evolution of the gain spectrum obtained using these equations as the


Fig. 5. Gain spectra calculated using analytical solutions of the coupled mode equations while varying the fiber length. Maximum depletion of the pump (at the central frequency) occurs for $L \approx 0.46 \mathrm{~km}$. Here, $\beta_{2}=-0.050 \mathrm{ps}^{2} / \mathrm{km}, \beta_{4}=2.5 \times$ $10^{-4} \mathrm{ps}^{4} / \mathrm{km}, \gamma=15 \mathrm{~W}^{-1} \mathrm{~km}^{-1}, P_{\mathrm{p} 0}=1 \mathrm{~W}$ and $P_{\mathrm{s} 0}=5 \mu \mathrm{~W}$.
fiber length increases while keeping $\beta_{2}$ and $\beta_{4}$ linked by Eq. (4). The depression in the gain occurring for longer fiber lengths results from back conversion related to the nonlinear phase mismatch in the strong depletion regime, which not only depends on pump power but also on signal and idler powers and distance along the fiber $z$, and is given by $\kappa(z)=\Delta \beta+\gamma\left(2 P_{\mathrm{p}}(z)-P_{\mathrm{s}}(z)-\right.$ $\left.P_{\mathrm{i}}(z)\right)[7,19]$. In Fig. 5 there is clearly a limit to the maximum gain that can be obtained at the central wavelength corresponding to the detuning $\Omega^{2}=$ $-6 \beta_{2} / \beta_{4}$ which is related to the maximum pump depletion possible. In fact, as can be seen in Eq. (5), pump is converted in signal and idler power while $\theta$ is positive. If $\theta$ is negative, signal and idler are converted back to pump power. It can be shown [19], that the pump power oscillates periodically along the fiber. An approximate expression for the fiber length that gives the maximum pump depletion is obtained in Appendix A:
$L_{\text {opt }} \approx \frac{1}{2 \gamma P_{0}} \ln \frac{16 P_{0}}{3 P_{\mathrm{s} 0}}$.
Here, $P_{0}$ is the total power which can also be written as $P_{0}=P_{\mathrm{p} 0}+P_{\mathrm{s} 0}$, where $P_{\mathrm{p} 0}$ and $P_{\mathrm{s} 0}$ are the input pump and signal powers in the fiber, respectively. For the conditions of Fig. 5, this results in $L_{\text {opt }}=0.46 \mathrm{~km}$. However, when using this optimum fiber length and the condition of flat gain obtained in the no-depletion approximation, Eq. (4), there appears a slight ripple in the spectrum for $L_{\text {opt }}$, as can be seen in Fig. 5. This happens because the nonlinear phase mismatch $\kappa$, used to derive Eq. (4), is no longer given by $\kappa=\Delta \beta+2 \gamma P_{P 0}$ as this is valid only in the no-depletion approximation. Clearly, the condition of flatness given by (4) is no longer valid and has to be modified. We have observed, after several simulations, that the ripple occurring using (4) and (6) depends on $P_{\mathrm{s} 0} / P_{\mathrm{p} 0}$. In this way we were able to obtain an empirical correction for condition (4) which is found to be
$\beta_{2}=-\sqrt{\frac{2}{3} \gamma P_{\mathrm{p} 0} \beta_{4}}\left[1-0.33\left(\frac{P_{\mathrm{s} 0}}{P_{\mathrm{p} 0}}\right)^{1 / 6}\right]$.
As also shown in Appendix A, the maximum flat gain obtained using Eqs. (6) and (7) is, to a good approximation, given by
$G_{\max }^{\text {flat }}(\mathrm{dB}) \approx 10 \log \left(\frac{2 P_{0}}{7 P_{\mathrm{s} 0}}\right)$.
The result is, however, always slightly less than the result of the analytic solution. More explicitly, for high values of the ratio $P_{\mathrm{s} 0} / P_{0}$ such as 0.1 , the maximum gain obtained by Eq. (8) is below the exact value calculated using Eqs. (6) and (7) and the analytic solution by 1.5 dB , but for lower, more realistic ratios, say $10^{-4}-10^{-6}$, the value given by Eq. (8) is only 0.5 dB below the exact value, which is a small error since the gains for this power ratio are above 30 dB .

The optimum length and the resulting maximum flat gain are key parameters of flat FOPAs. Eqs. (6) and (8) relate these design parameters with the input pump and signal powers in the fiber and the fiber nonlinear coefficient $\gamma$. We can also see in Fig. 5 that a trade-off exists between spectral flatness and gain bandwidth. Increasing the fiber length slightly beyond the optimum value for flatness, i.e., to $L=0.50 \mathrm{~km}$ here, results in a significantly larger $3-\mathrm{dB}$ bandwidth with a minimal increase in gain ripple, which may be an acceptable trade-off in a given system.

## 4. Conclusions

In this paper, we have demonstrated that the proper exploitation of the dispersion parameters $\beta_{2}$ and $\beta_{4}$ can result in ripple-free, broadband parametric gain spectra with 50 dB or more of gain and more than 40 nm of single-sided bandwidth. We have developed design rules for obtaining such gain spectra which are valid for any combination of realistic experimental parameters, supported by numerical simulations and analytical analysis. Important avenues for future work are the experimental demonstration of the predicted flat gain spectra, and the inclusion in the model of conditions to be found in a real DWDM system. Such conditions are the finite bandwidth of the pump, the effects of several simultaneous signal channels, sensitivity to polarization mode dispersion (PMD) and the degradation of amplifier performance due to random variations of parameter values along the length of the fiber [26].

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## Appendix A

In this section we explain the steps used to obtain Eq. (6). The coupled set of Eq. (5) can be solved analytically [ $19,24,25$ ]. To accomplish this we first recognize the invariants [19]:
$P_{0}=P_{\mathrm{p}}(z)+P_{\mathrm{s}}(z)+P_{\mathrm{i}}(z)=P_{\mathrm{p} 0}+P_{\mathrm{s} 0}$,
$\alpha=\frac{P_{\mathrm{s}}(z)-P_{\mathrm{i}}(z)}{P_{0}}=\frac{P_{\mathrm{s} 0}}{P_{0}}$,

$$
\begin{align*}
H= & \frac{1}{P_{0}^{2}}\left(4 P_{P}(z) \sqrt{P_{\mathrm{s}}(z) P_{\mathrm{i}}(z)} \cos \theta(z)\right. \\
& \left.-\left(\frac{\Delta \beta}{\gamma P_{0}}-1\right) P_{\mathrm{p}}(z) P_{0}-\frac{3}{2} P_{\mathrm{p}}^{2}(z)\right), \tag{A.3}
\end{align*}
$$

where $P_{0}$ is the total power in the fiber. Also the initial idler power is $P_{\mathrm{i}}(0)=P_{\mathrm{i} 0}=0$.

Using these invariants in the first of the coupled equations (5) we obtain
$\frac{\mathrm{d} \eta}{\mathrm{d} \xi}= \pm \sqrt{f(\eta)}$,
where $\eta=P_{\mathrm{p}}(z) / P_{0}$ and $\xi=\gamma P_{0} z$ are the normalized pump power and the normalized distance and

$$
\begin{align*}
f(\eta)= & 4 \eta^{2}\left[\left(1-\eta^{2}\right)-\alpha^{2}\right] \\
& -\left[H+\left(\frac{\Delta \beta}{\gamma P_{0}}-1\right) \eta+\frac{3}{2} \eta^{2}\right] \tag{A.5}
\end{align*}
$$

is a polynomial of fourth-order in $\eta$. It can be shown that Eq. (A.4) can be integrated so that the normalized pump power along the fiber is given by [24]
$\eta(\xi)=\frac{\eta_{2}-\eta_{0} \eta_{1} s n^{2}\left(\left.\frac{\xi-z_{0}}{z_{c}} \right\rvert\, k\right)}{1-s n^{2}\left(\left.\frac{\xi-z_{0}}{z_{c}} \right\rvert\, k\right)}$,
where $\eta_{1}, \eta_{2}, \eta_{3}$ and $\eta_{4}$ are the roots of the fourthorder polynomial (A.5) in ascending order. The
function $\operatorname{sn}(u \mid m)$ is the Jacobian elliptic function [27] and

$$
\begin{align*}
& \eta_{0}=\frac{\eta_{3}-\eta_{2}}{\eta_{3}-\eta_{1}},  \tag{A.7}\\
& k^{2}=\frac{\eta_{3}-\eta_{2}}{\eta_{3}-\eta_{1}} \frac{\eta_{4}-\eta_{1}}{\eta_{4}-\eta_{2}}, \tag{A.8}
\end{align*}
$$

$z_{c}=\frac{2}{\left(\frac{7}{4}\left(\eta_{3}-\eta_{1}\right)\left(\eta_{4}-\eta_{2}\right)\right)^{1 / 2}}$,
$z_{0}=z_{c} F\left(\sin ^{-1}\left(\sqrt{\frac{P_{\mathrm{p} 0}-P_{0} \eta_{2}}{\eta\left(P_{\mathrm{p} 0}-P_{0} \eta_{1}\right)}}\right), k\right)$
where $F(u, m)$ is the elliptic integral of first kind associated with the complete elliptic integral $K(m)$ [27].

The function $\operatorname{sn}(u \mid m)$ is periodic with period of $4 K(m)$ so that the normalized pump power has a spatial period $\xi_{\mathrm{p}}=z_{\mathrm{c}} 2 K(k)$. Therefore the normalized pump experiences maximum depletion for half of this period, which occurs when the phase $\theta=0$ as discussed before. This defines our optimum fiber length
$z_{\text {opt }}=\frac{z_{\mathrm{c}}}{\gamma P_{0}} K(k)$.
Eq. (A.11) can be further simplified if we evaluate the invariants (at $z=0$ ) for the central frequency, defined to be the frequency where the minimum of $\kappa$ occurs. As we have already shown this frequency corresponds to a detuning $\Omega^{2}=-6 \beta_{2} / \beta_{4}$. Setting $\kappa_{\text {min }}=0$ at this frequency (flat spectrum condition), we have $\Delta \beta=-2 \gamma P_{\mathrm{p} 0}$ so that:

$$
\begin{align*}
& \frac{\Delta \beta}{\gamma P_{0}} \approx \frac{\Delta \beta}{\gamma P_{\mathrm{p} 0}}=-2,  \tag{A.12}\\
& H \approx \frac{3}{2},  \tag{A.13}\\
& \alpha=\frac{P_{\mathrm{s} 0}}{P_{0}} . \tag{A.14}
\end{align*}
$$

We note here that the use of the flat spectrum condition in the no-depletion regime is valid since we are evaluating the invariant at the fiber input where depletion certainly does not occur.

Using Eqs. (A.12)-(A.14) we can find the roots of Eq. (A.5) for the central frequency. The roots will depend only on $\alpha$. The quantities $z_{c}$ and $k^{2}$ can then be expanded in terms of $\alpha$ (since generally $\alpha \ll 1$, or in other words the input signal power is very small compared with the total power). The Taylor expansions of these quantities read:
$z_{\mathrm{c}} \approx 1$,
$k^{2} \approx 1-3 \alpha$.
Now we use the asymptotic form of $K(k)$ for $k$ approaching unity [27]
$K(k) \approx \frac{1}{2} \ln \frac{16}{1-k^{2}} \approx \frac{1}{2} \ln \frac{16}{3 \alpha}$.
In this way $L_{\text {opt }}$ is given by
$L_{\mathrm{opt}}=z_{\mathrm{opt}} \approx \frac{1}{2 \gamma P_{0}} \ln \frac{16 P_{0}}{3 P_{\mathrm{s} 0}}$.
It is now possible to evaluate the pump power at the length $L_{\text {opt }}$. Since at $L_{\text {opt }}$ the $s n^{2}(u \mid m)$ of Eq. (A.6) is zero, we have
$P_{\mathrm{p}}\left(L_{\text {opt }}\right)=P_{0} \eta_{2} \approx \frac{3}{7} P_{0}$,
where $\eta_{2}$ expanded in a Taylor series reads $\eta_{2} \approx(3 / 7)+(3 / 8) \alpha^{2}$.

In this way, the signal power along the fiber, which is given by
$P_{\mathrm{s}}(z)=\frac{P_{0}(1+\alpha)-P_{\mathrm{p}}(z)}{2}$,
becomes at $L_{\mathrm{opt}}$

$$
\begin{align*}
P_{\mathrm{s}}\left(L_{\mathrm{opt}}\right) & \approx \frac{P_{0}(1+\alpha)-(3 / 7) P_{0}}{2} \\
& =\frac{P_{0}}{2}\left(\frac{4}{7}+\alpha\right) . \tag{A.21}
\end{align*}
$$

It follows that an estimate of the maximum gain at the central frequency (at $L_{\mathrm{opt}}$ ) is given by

$$
\begin{align*}
G_{\max } & =10 \log \left(\frac{P_{\mathrm{s}}\left(L_{\mathrm{opt}}\right)}{P_{\mathrm{s} 0}}\right) \\
& \approx 10 \log \left(\frac{2}{7 \alpha}+\frac{1}{2}\right) \\
& \approx 10 \log \left(\frac{2 P_{0}}{7 P_{\mathrm{s} 0}}\right) . \tag{A.22}
\end{align*}
$$

## References

[1] M.E. Marhic, N. Kagi, T.-K. Chiang, L.G. Kazovsky, Opt. Lett. 21 (1996) 573.
[2] J. Hansryd, P.A. Andrekson, IEEE Photon. Technol. Lett. 13 (2001) 194.
[3] M.E. Marhic, Y. Park, F.S. Yang, L.G. Kazovsky, Opt. Lett. 21 (1996) 1354.
[4] L. Provino, A. Mussot, E. Lantz, T. Sylvestre, H. Maillotte, in: Technical Digest of the Optical Fiber Communication Conference, Optical Society of America, Washington, DC, 2002, p. 125.
[5] K.K.Y. Wong, M.E. Marhic, K. Uesaka, L.G. Kazovsky, in: Technical Digest of the Optical Fiber Communication Conference, Optical Society of America, Washington, DC, 2002, p. 129.
[6] M.C. Ho, K. Uesaka, M.E. Marhic, Y. Akasaka, L.G. Kazovsky, IEEE J. Lightwave Technol. 19 (2001) 977.
[7] K. Inoue, T. Mukai, Opt. Lett. 26 (2001) 10.
[8] J.L. Blows, S.E. French, Opt. Lett. 27 (2002) 491.
[9] C.J.S. de Matos, D.A. Chestnut, P.C. Reeves-Hall, J.R. Taylor, Opt. Lett. 26 (2001) 1583.
[10] D.A. Chestnut, C.J.S. de Matos, J.R. Taylor, J. Opt. Soc. Am. B 19 (2002) 1901.
[11] J.M. Chávez Boggio, F.A. Callegari, S. Tenenbaum, H.L. Fragnito, J.B. Rosolem, M.R.X. de Barros, in: Technical Digest of the Optical Fiber Communication Conference, Optical Society of America, Washington, DC, 2002, p. 636.
[12] J. Hansryd, P. Andrekson, M. Westlund, J. Li, P.-O. Hedekvist, IEEE J. Sel. Top. Quantum Electron. 8 (2002) 506.
[13] S.E. French, J.L. Blows, in: N. Jolley, J.D. Minelly, Y. Nakano (Eds.), Optical Amplifiers and their Applications, OSA Trends in Optics, Photonics Series, vol. 60, Optical Society of America, Washington, DC, 2001, p. 88.
[14] M.E. Marhic, M.C. Ho, L.G. Kazovsky, IEEE J. Lightwave Technol. LT-17 (1999) 210.
[15] C.J. McKinstrie, S. Radic, A.R. Chraplyvy, IEEE Sel. Top. Quantum Electron. 8 (2002) 538.
[16] J.M. Chávez Boggio, S. Tenenbaum, H.L. Fragnito, J. Opt. Soc. Am. B 18 (2001) 1428.
[17] A. Vatarescu, IEEE J. Lightwave Technol. LT-5 (1987) 1652.
[18] R.H. Stolen, J.E. Bjorkholm, IEEE J. Quantum Electron. QE-18 (1991) 824.
[19] G. Cappellini, S. Trillo, J. Opt. Soc. Am. B 8 (1991) 824.
[20] M. Yu, C.J. McKinstrie, Phys. Rev. E 52 (1995) 1072.
[21] G.P. Agrawal, Nonlinear Fiber Optics, second ed., Academic Press, San Diego, CA, 1995.
[22] M.E. Marhic, in: Technical Digest of the Optical Fiber Communication Conference, Optical Society of America, Washington, DC, 2003, p. 564.
[23] M.E. Marhic, K.K.Y. Wong, M.C. Ho, L.G. Kazovsky, Opt. Lett. 26 (2001) 620.
[24] Y. Chen, J. Opt. Soc. Am. B 6 (1989) 1986.
[25] Y. Chen, A.W. Snyder, Opt. Lett. 14 (1989) 87.
[26] M. Karlsson, J. Opt. Soc. Am. B 15 (1998) 2269.
[27] I.S. Gradshteyn, I.M. Ryzhik, Table of Integrals, Series, and Products, Corrected and Enlarged Edition, Academic Press, San Diego, CA, 1980.


[^0]:    Project
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