The Capacity of Fiber-Optic Communication Systems

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Abstract: We present a capacity estimate of fiber-optic communication systems limited by fiber nonlinearity. The analysis reveals that a capacity of ∼5 bits/s/Hz in a single polarization for transmission over 2000 km is possible using advanced technologies.

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1. Introduction

The information carrying capacity of a communication channel was first considered by Shannon in 1948 [1] who calculated the capacity of a memoryless channel with additive white Gaussian noise (AWGN) for a given signal-to-noise ratio (SNR). It was later shown by Gordon [2],[3] that amplified spontaneous emission (ASE) can be represented by AWGN fields, making possible to use Shannon's theory to optically amplified systems.

The application of Shannon's theory to the optical 'fiber channel' faces many challenges. The most important is that three phenomena are *simultaneously* at play in fibers: *amplified spontaneous emission*, *chromatic dispersion*, and the (instantaneous) *Kerr fiber nonlinearity* (see Fig. 1a). Estimations of the 'fiber capacity' that include fiber nonlinearity have relied on a variety of assumptions such as weak nonlinearity [4], low dispersion [5] or heuristic [6] and information rates [7] approaches. In these studies, no connections are made to modulation, constellations and nonlinearity compensation.

Here, we present a method for evaluating a conservative estimate of the 'fiber channel' capacity by using a modulation with compact spectrum, multi-level amplitude and phase modulations, high-speed pseudo-linear transmission, reverse nonlinear propagation combined with pre-distortion at the transmitter and coherent detection. The analysis captures all instantaneous fiber Kerr nonlinearities, including nonlinear signal-noise interactions. Using such advanced technologies, a capacity lower bound estimate for a 2000-km wavelength-division multiplexing (WDM) transmission is performed.

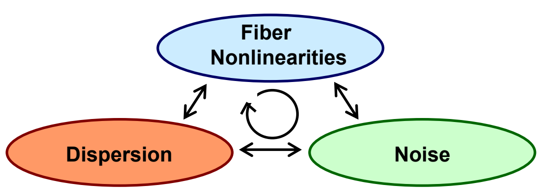
2. Fiber Propagation

The evolution of the optical field $E(z,t)$ in the presence of instantaneous Kerr nonlinearity in fibers is given by [8]

$$\frac{\partial E}{\partial z} + \frac{i}{2}\,\beta_2(z)\,\frac{\partial_2 E}{\partial t^2} = i\,\gamma(z)\,|E|^2 E + i\,n(z,t), \tag{1}$$

where $\beta_2(z)$ and $\gamma(z)$ are the fiber dispersion and nonlinear coefficient [8] as a function of the propagation distance in the fiber. The stochastic term $n(z,t)$ describes the AWGN. It is defined by its autocorrelation $< n(z,t)\, n^*(z',t') > = \mathrm{n_{sp}} h\nu_s \alpha\, \delta(z-z', t-t')$ where h is the Planck constant, δ the Dirac functional, and other parameters are defined in the Table of Fig. 1b. Dispersion slope is neglected here as it has less impact than dispersion when operating away from the zero-dispersion wavelength. Alternatively, dispersion slope can also be engineered to zero. Distributed amplification compensating exactly for fiber loss is assumed in order to maximize the optical signal-to-noise ratio (OSNR). The delivered OSNR is given by $P_\mathrm{s}/(2N_\mathrm{ASE}B_\mathrm{Ref})$ where P_s is the signal power per WDM channel, B_Ref is the reference bandwidth entering the definition of the OSNR (12.5 GHz) and N_ASE is the spectral density of the noise per polarization. N_ASE is given by $n_\mathrm{sp} h\nu_s \alpha L$ where L is the transmission length.

(a)



(b)

Number of spans	20
Span length	100 km
Dispersion $D\,(=-2\pi c\beta_2/\lambda_s^2)$	17 ps/(nm-km)
Loss coefficient α	0.2 dB/km
Nonlinear coefficient γ	1.27 (W-km)$^{-1}$
Symbol rate S	100 Gbaud
Channel spacing B	125 GHz
Mux and demux bandwidth	100 GHz
Optical frequency of central channel ν_s	193.55 THz
Spontaneous emission factor $\mathrm{n_{sp}}$	1

Fig. 1. Propagation effects and system parameters: a) The three distributed phenomena at play for the optical fiber channel and b) Parameters of the example system considered in this paper.

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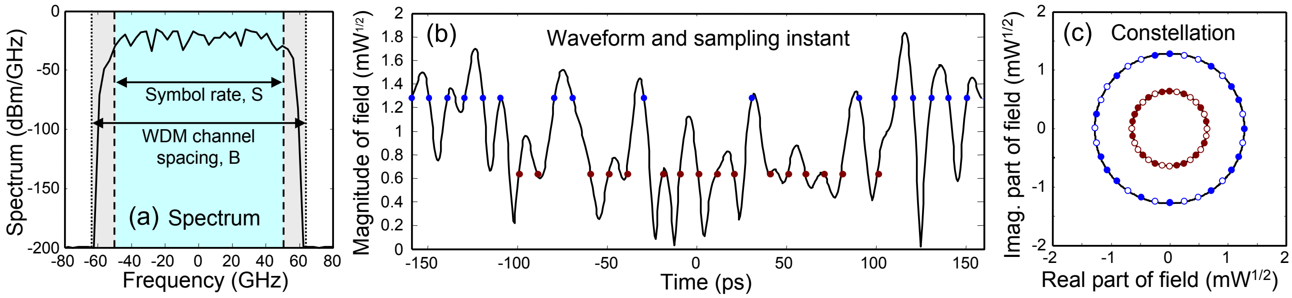

Fig. 2. Example of constellation and field used in this study: a) Raised cosine spectrum, b) waveform and sampling instants and c) symbols and constellation. To ease visualization, a small alphabet of 32 symbols (full circles) over only 2 rings on a low-resolution grid (open circles) is represented.

Compensation of intra-channel nonlinearities is applied at the transmitter through reverse propagation [9] of the channel in the absence of noise. This removes the channel memory associated with the pattern dependence of signal-signal intra-channel nonlinearities [10]. A single state of polarization is considered in this study. For optical dispersion compensation, we use a singly-periodic dispersion map optimized *in the absence* of intra-channel nonlinearity compensation. The dispersion compensating elements are considered as low-loss linear elements. The dispersion pre-compensation is -1050 ps/nm, the residual dispersion per span is 20 ps/nm (applied every 100 km) and the dispersion is brought to zero at the receiver. We use such a dispersion map to minimize the impact of imperfect compensation in practical implementations of intra-channel nonlinearities compensation. The noise co-propagates with the signal, capturing all signal-noise nonlinear interactions.

3. Modulation and Constellations

Achieving high-capacity transmission requires multi-level symbol constellations with compact spectra. The modulation considered in this study uses Nyquist signals having 'box-like' spectra with a square-root raised cosine shape (in the optical field) and a roll-off of 20% (see Fig. 2a) [11]. The optical multiplexer and demultiplexer transfer functions are identical and match the square-root raised cosine signal spectrum. The modulation is free from inter-symbol interference (ISI) as seen in Figs. 2b and 2c. The raised cosine roll-off is chosen to reduce the large memory in the time domain associated with perfectly square spectrum modulation using the 'sinc' temporal function [11]. The constellation uses a concentric N-ring structure in field (amplitude shift-keying, N-ASK) with equal amplitude spacing and random phase (phase shift-keying, PSK) on a high-resolution angular grid. For each symbol, amplitude and phase are randomly chosen within the underlying constellation, with equal population of each ring. The actual number of phase states needed in practical implementation follows from the capacity results and the coding used.

4. Information Theory and Channel Capacity

The channel capacity for a specified channel input alphabet, when X is a random input giving rise to the random channel output Y, is given by the relation [1, 12]

$$\begin{aligned} C/B &= \iint p_{Y,X}(y,x) \log_2 \frac{p_{Y,X}(y,x)}{p_Y(y) p_X(x)} \, dy \, dx \\ &= -\int p_Y(y) \log_2 p_Y(y) \, dy + \iint p_{Y,X}(y,x) \log_2 p_{Y|X}(y|x) \, dy \, dx \\ &= H(Y) - H(Y|X) \, , \end{aligned} \tag{2}$$

where B is the channel spacing (see Fig. 2a), $p_{Y,X}$ is the joint probability density function (pdf) of the received and transmitted signals, Y and X, while p_Y and p_X are the marginal densities [13]. The conditional pdf of Y given X is $p_{Y|X}$. The functions $H(Y)$ and $H(Y|X)$ are referred to as the entropy of Y and the entropy of Y conditioned on X, respectively. For the numerical evaluation of fiber capacity, we treat the channel as a discrete memoryless channel (DMC) [1, 12, 14]. Using such a model is in part motivated by the complete removal of the memory associated with signal-signal intra-channel nonlinearities, as described in Sec. 2. Nevertheless, using a DMC model result in a lower bound capacity estimate. The discretized version of Eq. (2) is applied for the case of a concentric ring input alphabet by summing up the entropies on the RHS of Eq. (2). Full knowledge of the received field, corresponding to an ideal coherent receiver, is assumed to obtain Y.

5. Capacity Results

We first evaluated the capacity of our constellation structure in the absence of fiber nonlinearities (Fig. 3a). Shannon's capacity, given by $C = B \, \log_2(1 + \text{SNR})$, is also shown for comparison. In Shannon's equation (2), the SNR (not to be confused with the SNR used in optical communication) is the ratio of the energy per bit E_b to the noise per bit N_0 [11]. Shannon's SNR relates to the OSNR by $\text{OSNR} = S/(2\,\text{B}_{\text{ref}})\,\text{SNR}$, where S is the signal symbol rate and B_{ref} is the 12.5-GHz reference bandwidth entering the definition of the OSNR. As seen in Fig. 3a, as the SNR increases, a larger number

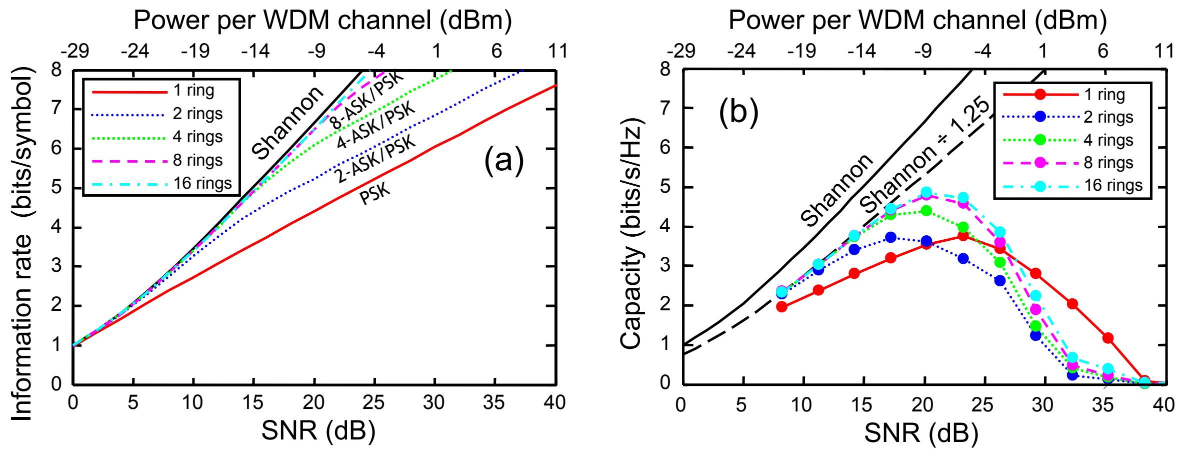

Fig. 3. Capacity of a linear and nonlinear system: a) Capacity as a function of SNR in the absence of fiber nonlinearity, b) Capacity as a function of SNR for the 2000-km system considered for various number of rings. All fiber nonlinearities are included.

of rings is necessary to reach the Shannon capacity. This originates from the fact that, at high SNR, the capacity is better maximized by filling the complex plane than adding symbols on a single or a few rings. Figure 3a can be used to determine the gain in capacity possible by moving from a single ring, i.e. pure PSK, to multiple rings, i.e. N-ASK/PSK (N integer), at powers below the nonlinearity threshold.

Results for nonlinear transmission in the WDM system considered are presented in Fig. 3b. We use a total of 2048 symbols, selected as described in Sec. 3. The capacity in Fig. 3b is expressed in bits/s/Hz and is obtained by dividing the information rate, expressed in bits/symbol, of the central channel (out of the five simulated WDM channels) by the ratio B/S (= 1.25 (s Hz)/symbol). This reduction in capacity is due to limitations in spectral packing associated to the 'practical' raised cosine modulation used and is not present for square spectrum modulation. This spectral packing factor reduces the capacity relative to Shannon (as seen at low SNRs of $\sim$10 dB) but follows the Shannon capacity divided by the B/S ratio (long-dashed curve in Fig. 3b). The capacities for different numbers of rings peak around $\sim$20 dB SNR and reach $\sim$5 bits/s/Hz for 16 rings. For this specific system, the capacity is limited by cross-phase modulation (XPM). Computation with four different noise seeds and four different sets of random phases lead to variations in maximum capacity of $\pm$0.3 bits/s/Hz.

6. Conclusion

We presented a general method to evaluate the fundamental capacity of fiber-optic communication systems. We considered a 2000-km transmission line and found a fiber capacity of $\sim$5 bits/s/Hz. The method presented can be applied to different transmission lines to assess the ultimate capacity achievable using advanced electronic and optical technologies.

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